CES Technology and Business Cycle Fluctuations[1]

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Motivation

- Cobb-Douglas Production is widely assumed in the RBC, DSGE and growth literatures
- Mounting evidence that the capital-labour ratio, factor price ratio elasticity $\sigma << 1$
- See literature and evidence on US Labour share
- Old literature on CES PF going back to [Solow(1956)] and [Arrow *et al.*(1961)]
- Implementing a CES PF in a RBC or DSGE model is not straightforward! – **problem of normalization** (see, for example, [La Grandville(1989)], [León-Ledesma *et al.*(2010)], [Cantore *et al.*(010a)] and [Cantore and Levine(2010)])

US Labour Share

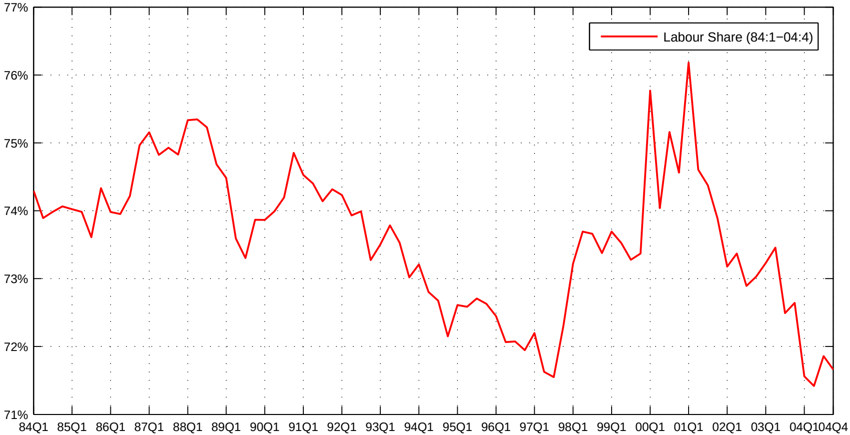


Figure: US Labour Share (Source: Department of Labor, U.S. Bureau of Labor Statistics)

Main Results

- We confirm decisively the importance of CES rather than Cobb-Douglas (CD) production functions for explaining business cycles in our DSGE model estimated in DYNARE by Bayesian-Maximum-Likelihood methods using US data.
- A marginal likelihood (ML) race assuming equal prior model probabilities, CES beats the CD production function with posterior model probabilities 0.999988 : 0.000012.
- The ML improvement is matched by the ability of the CES model to get closer to the data in terms of second moments.
- A comparison with a DSGE-VAR further confirms the ability of the CES model to reduce model misspecification.
- We estimate $\sigma = 0.36$.
- The main message then for DSGE models is that we should dismiss once and for all the use of CD for business cycle analysis.

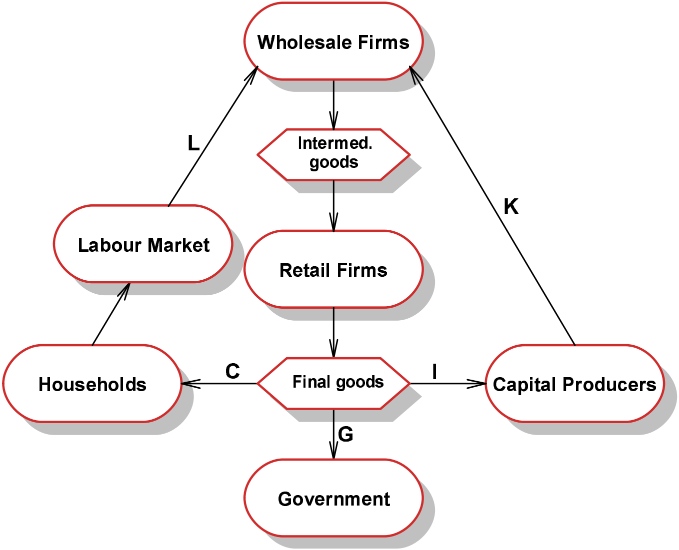
Plan of Talk

- Model Structure
- The Normalization Problem
- Bayesian ML Estimation Results
- Validation (Second Moments, DSGE-VAR, IRFs)
- Variance Decomposition
- Shortcomings and Future Research

The Model: From RBC to NK

1. **An RBC Core**
   - Household make an intertemporal utility-maximizing choice of consumption and labour supply over time subject to a budget constraint
   - Firms produce output according to a production technology and choose labour and capital inputs to minimize cost
   - Labour, output and financial markets clear
   - Add investment costs
2. Add monopolistic competition in wholesale or retail market and price stickiness (Calvo contracts)
3. Ditto in labour market (LM) and wage stickiness
4. Add a nominal interest rate Taylor rule with persistence
5. Arrive at the **Core SW-type NK Model**
6. Then move on to financial frictions (banking sector), search-match LM frictions, non-trivial government budget constraint with distortionary taxes and fiscal rule, openness etc

Model Structure



The Normalization Problem

- The CES production function is given by

$$\begin{aligned} Y_t &= \left[\alpha_k (ZK_t K_t)^{\psi} + \alpha_n (ZN_t N_t)^{\psi}\right]^{\frac{1}{\psi}} \; ; \psi \neq 0 \; \& \; \alpha_k + \alpha_h \neq 1 \\ &= (ZK_t K_t)^{\alpha_k} (ZN_t N_t)^{\alpha_n} \; ; \psi \to 0 \; \& \; \alpha_k + \alpha_h = 1 \end{aligned} \tag{1}$$

  where Y_t, K_t, N_t are output, capital and labour inputs respectively at time t, ψ is the substitution parameter and α_k and α_n are sometimes referred as 'distribution parameters'.

- ZK_t and ZN_t capture capital-augmenting and labour-augmenting technical progress respectively.
- Calling σ the elasticity of substitution between capital and labour with $\sigma \in (0, +\infty)$ and $\psi = \frac{\sigma - 1}{\sigma}$ then $\psi \in (-\infty, 1)$. When $\sigma = 0 \Rightarrow \psi = -\infty$ we have the Leontief case, whilst when $\sigma = 1 \Rightarrow \psi = 0$, collapses to the usual Cobb-Douglas case.

What is the Problem?

- Basically α_k and α_n cannot be estimated as they depend on the units of output and inputs chosen and therefore are not pure numbers.
- Let α and $1-\alpha$ be the capital and labour shares in the balanced growth path (bgp) steady state. Then using the bgp steady state foc for factor inputs we can obtain

$$\alpha_k = \alpha \left(\bar{Y}_t/(ZK\bar{K}_t)\right)^{\psi}$$

$$\alpha_n = (1-\alpha)\left(\bar{Y}_t/(\overline{ZN}_t N)\right)^{\psi}$$

- Now these dimensional parameters are expressed in terms of other endogenous variables Y, N and K which themselves are functions of $\theta \equiv [\sigma, \psi, \pi, \delta, \cdots]$. Therefore $\alpha_n = \alpha_n(\alpha, \theta)$, and $\alpha_k = \alpha_k(\alpha, \theta)$ which expresses why we refer to this procedure as **reparameterization** – easily set up in DYNARE.

BML Estimation

- **7 Observables:** Log differences GDP, consumption, investment, real wage and levels of inflation, nominal interest rate and hours worked
- **8 Shocks:** 2 technology, investment, government exp., preference, price and wage mark-ups, monetary
- **Results** (from DYNARE) ▸ Estimation

Model	σ	Technology shocks	LL	Diff with CD
CD	1	ZN	-469.13	0
Calib. CES	0.4	ZK & ZN	-458.54	10.58
CES1	0.33	ZN	-459.23	9.89
CES2	0.36	ZK & ZN	-460.24	8.88

Table: Log Marginal Likelihood comparison between CD and CES specifications

Validation

- **Second Moments**
  - Volatility – Standard Deviations
  - Co-Movement – Cross Correlations
  - Persistence - Autocorrelation
- Follow [Del Negro and Schorfheide(2004)] and compare with a Benchmark by constructing a hybrid combination of an unrestricted VAR and the VAR implied by the estimated'**DSGE-VAR**'.
  - The hyper-parameter λ define extent to which the DSGE imposes restrictions on the VAR.
  - If λ is small the prior is diffuse. When $\lambda = \infty$, we obtain a VAR approximation of the log-linearized DSGE model.
- Compare **IFRs** of models with those of the DSGE-VAR

Validation: Second Moments

			Standard Deviation				
Model	Y	C	I	W/P	Π	R_n	h
Data	0.59	0.53	1.80	0.60	0.25	0.64	2.47
CD	0.93	0.66	2.15	0.65	**0.37**	0.43	5.56
CES1	**0.82**	**0.56**	**1.76**	**0.59**	0.47	**0.54**	**5.51**
CES2	**0.82**	**0.56**	1.78	**0.59**	0.46	0.53	5.58
			Cross-correlation with Output				
Data	1.00	0.60	0.63	-0.10	-0.14	0.14	-0.22
CD	1.00	**0.51**	0.80	0.33	**-0.20**	-0.30	0.13
CES1	1.00	0.43	**0.73**	**-0.05**	0.08	**-0.07**	**0.10**
CES2	1.00	0.44	0.74	-0.04	0.09	**-0.07**	**0.10**

Table: Selected 'Unconditional' Second Moments of the Model Variants

Validation: Autocorrelations

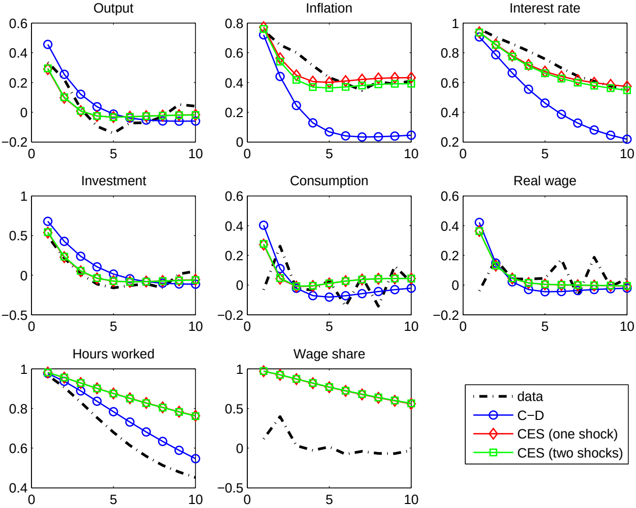


Figure: Autocorrelations of Observables in the Actual Data and in the Estimated Models

Validation: DSGE-VAR

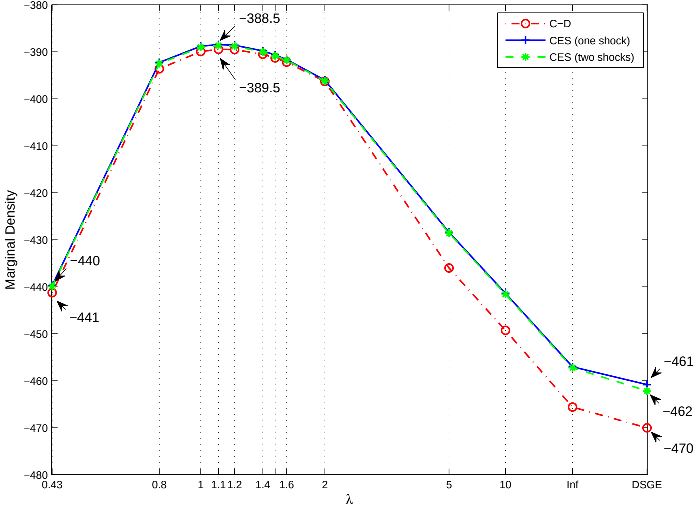


Figure: Marginal Likelihood as a Function of λ

Validation: IRFs

- Clearly the most important difference comes from fluctuations in factor shares under the CES specification.
- Fluctuations of shares translate as well in different IRFs of interest rate and wage in the two models.
- I - shock: both, wage and interest rate, present a more sluggish response to an investment specific shock under CES and, as a result, a quite different response of consumption and inflation.
- L-aug shock: we find that overall the discrepancy between VAR and DSGE is relatively smaller under the CES production assumption. This suggests that the DSGE model misspecification is larger with the CD production than with the CES.

▸ IRF figures

IRFs - cont'd

- If we study carefully the responses to the other shocks, we can generally find the similar conclusion that CES helps reduce the discrepancy although the IRFs to the investment-specific shock are the exception.
- To sum up, there also exists some evidence from IRFs in favour of the CES assumption in DSGE models, but the evidence from the IRFs is not as strong as that obtained by comparing the moments and the marginal likelihood comparison amongst models which more clearly reject the CD specification.

Variance Decomposition

- The underlying sources of fluctuations at various horizons (explained by the CES model). ▸ VD figure
- Movements in GDP primarily driven by markup shocks and the exogenous spending shock (consistent with CD results).
- Policy shock is by far the most determinant s-r influence to the nominal interest rate. Wage markup becomes dominant in the l-r.
- Inflation fluctuations are mostly explained by the investment shock (short-medium run) but the main l-r driving factor becomes the wage markup shock.
- Wage markup shock clearly dominates behind the l-r movements in hours worked.
- The rest are similar to those obtained from CD (not shown).
- Overall significant impacts brought by CES assumption on contributions from investment, markup, gvt. spending shocks.

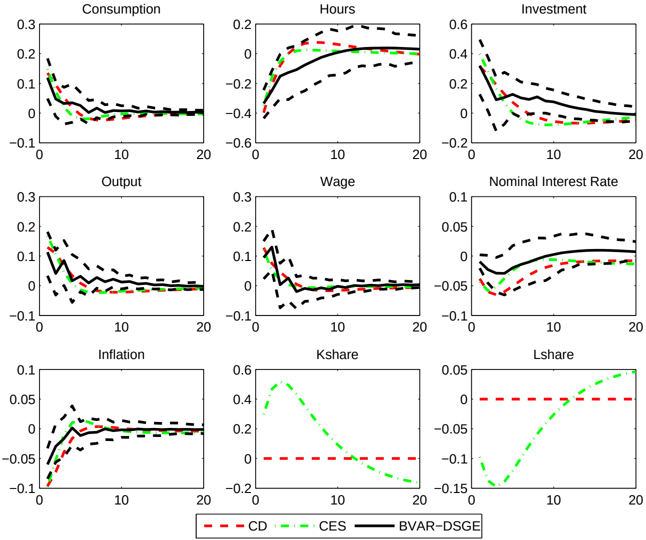
Shortcomings and Future Research

- A major concern in terms of model misspecification is in the second moments involving wages and hours. For example both CD and CES models fail miserably in reproducing the negative correlation between output and hours; furthermore the CES model produces far too much persistence in hours.
- A low capital-labour substitutability is crucial for understanding unemployment persistence ([Rowthorn(1999)]).
- Suggests that future research should introduce search-match frictions and unemployment alongside CES production.
- Cannot have both labour- and capital-augmenting technical change with CES – raises an obstacle to the prospect of unifying business cycle analysis with long-term endogenous growth based on CES . But see [León-Ledesma and Satchi(2010)] for a possible resolution.

DSGE Modelling and Policy Analysis Using DYNARE

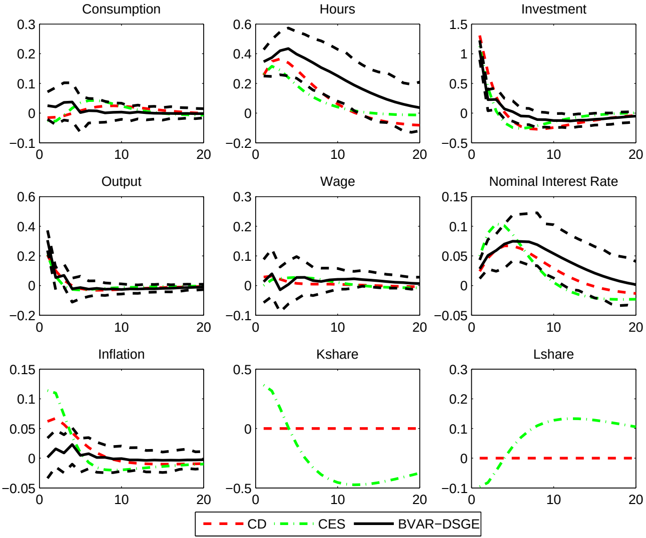
1. **Setting up DSGE Models in Dynare**.
2. **Estimation of DSGE Models in Dynare**.
    - Asymmetric Information Assumptions
    - Symmetric Information Assumption
3. **Identification**
4. **Model Validation**
5. **Policy Exercises.**
    - Ex Ante Optimal Policy
    - Optimal Timeless Policy
    - Time Consistent Policy
    - Optimized Simple Rules
    - All above with Perfect and Imperfect Information
    - Robust Policy Design
6. **Course and Dynare Software** available by end of 2011

Validation: IRFs Labour-augmenting shock



Validation: IRFs Investment-specific shock

return



Variance Decomposition

return

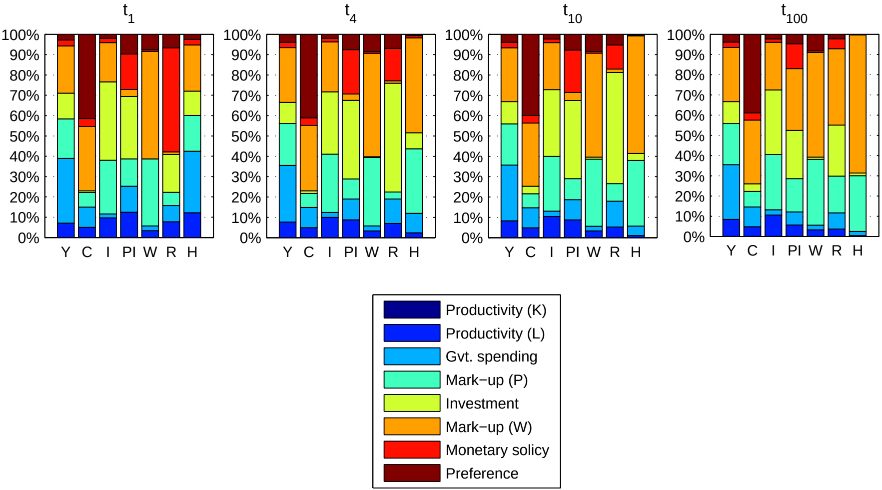


Figure: Forecast Error Variance Decomposition (CES)

parameter	prior mean	post. mean CD (SW07)	post. mean CES	5% CES	95% CES	Prior	pstdev CES
ρ_{ZL}	0.5	0.9004 (0.95)	0.8919	0.8386	0.9476	beta	0.2
ρ_{ZK}	0.5	N/A (N/A)	**0.5120**	0.1729	0.8790	beta	0.2
ρ_G	0.5	0.9561 (0.97*)	0.9297	0.9096	0.9533	beta	0.2
ρ_{ZI}	0.5	0.7162 (0.71)	0.6992	0.5397	0.8617	beta	0.2
ρ_P	0.5	0.9239 (0.89*)	0.9566	0.9235	0.9908	beta	0.2
ρ_W	0.5	0.9046 (0.96*)	0.9528	0.9142	0.9912	beta	0.2
ρ_B	0.5	0.6324 (N/A)	0.6057	0.3109	0.9183	beta	0.2
ε_{ZL}	0.1	0.5735 (0.45)	0.5743	0.4899	0.6532	invg	2.0
ε_{ZK}	0.1	N/A (N/A)	**0.0853**	0.0238	0.1612	invg	2.0
ε_G	0.5	2.4634 (0.53*)	2.4958	2.1909	2.8166	invg	2.0
ε_{ZI}	0.1	3.6743 (**0.45**)	**2.4553**	1.2221	3.9710	invg	2.0
ε_P	0.1	0.1136 (0.14*)	**0.1846**	0.1169	0.2486	invg	2.0
ε_W	0.1	0.3835 (0.24*)	0.4187	0.2769	0.5577	invg	2.0
ε_M	0.1	0.1585 (0.24)	0.1556	0.1318	0.1778	invg	2.0
ε_B	0.1	1.4830 (N/A)	**1.3850**	0.9620	1.7984	invg	2.0

Table: Posterior results for the exogenous shocks

parameter	prior mean	post. mean CD (SW07)	post. mean CES	5% CES	95% CES	Prior	pstdev CES
σ	1	1 (1)	**0.3622**	0.2391	0.4755	gamma	1
ϕ	0.5	0.9002 (0.54)	0.9215	0.8725	0.9738	beta	0.15
ϕ^X	2	2.5530 (2.87)	**1.3515**	0.5	2.2683	norm	1.5
σ_c	1.5	2.0158 (1.38*)	**2.1431**	1.6078	2.6755	norm	0.3750
ϱ	0.5	0.5061 (1.83*)	**0.2785**	0.1	0.4398	beta	0.20
χ	0.7	0.5092 (0.71*)	**0.4065**	0.2597	0.5540	beta	0.1
ξ_w	0.5	0.5632 (0.7)	0.5093	0.4233	0.5950	beta	0.1
ξ_p	0.5	0.7173 (0.66)	**0.6171**	0.5367	0.6994	beta	0.1
γ_w	0.5	0.4948 (0.58)	0.4903	0.2459	0.7259	beta	0.15
γ_p	0.5	0.2215 (0.24)	0.2749	0.0929	0.4478	beta	0.15
α	0.3	0.2239 (0.19)	0.2615	0.1985	0.3253	norm	0.05
α_π	1.5	2.1699 (2.04)	**2.3108**	1.9897	2.6187	norm	0.25
α_r	0.75	0.8221 (0.81)	0.8146	0.7772	0.8520	beta	0.1
α_y	0.25	0.0407 (0.08)	0.0710	0.0110	0.1254	norm	0.05
conspie	0.625	0.5201 (0.78)	0.5116	0.4461	0.5774	gamma	0.1
ctrend	0.4	0.4730 (0.43)	0.4756	0.4432	0.5072	norm	0.1

Table: Posteriors results for model parameters

▸ return

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