An Estimated DSGE Model of the Indian Economy

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NIPFP Research Agenda

• Build a Model of the Indian Economy that is ‘fit for purpose’
  • Micro-founded and more immune from the Lucas Critique
  • Incorporates important features of Emerging Economies and the Indian Economy in particular
  • From closed to open
  • Empirically based to enable quantitative analysis.
  • Hence DSGE Model estimated by Bayesian-Maximum-Likelihood Methods with calibrated values or estimates from microeconometric studies as priors

• Compare the welfare outcome of Taylor-type simple rules rules with existing policy
The Work So Far

- This paper: estimates a closed economy model with financial and labour market frictions and formal/informal sectors
- Model fitted to data on GDP, CPI inflation, Investment and a Nominal Interest Rate
- A paper on the Open Economy: [Batini et al. (2010a)], drawing on [Batini et al. (2007)] and [Batini et al. (2009)]
- Related paper: [Anand et al. (2010)]
Modelling Methodology

- Build up the model in stages
- Model 1: A standard NK model fitted to both Indian and US Data
- Model 2: Add financial frictions
- Model 3: Add informal sector with labour market frictions

Questions:
- What is different about the Indian (compared with the US) Economy?
- Do the new features improve the fit of the model?
- Does Bayesian ML Estimation in Model 3 provide information about the size of the informal sector?
The Models: From RBC to NK

• An RBC Core
  • Household make an intertemporal utility-maximizing choice of consumption and labour supply over time subject to a budget constraint
  • Firms produce output according to a production technology and choose labour and capital inputs to minimize cost
  • Labour, output and financial markets clear
  • Add investment adjustment costs

• Add monopolistic competition in retail market and price stickiness
• Add an interest rate Taylor rule with persistence
• Arrive at the standard NK Model
A Calvo-Type Interest Rate Taylor Rule

- Following [Levine et al. (2007)] and [Gabriel et al. (2009)], we model monetary policy in a very general way by formulating a Calvo-type forward-backward interest rate rule:

\[
\log \left( \frac{1 + R_{n,t}}{1 + R_n} \right) = \rho \log \left( \frac{1 + R_{n,t-1}}{1 + R_n} \right) + \theta \log \frac{\Theta_t}{\Theta} + \phi \log \frac{\Phi_t}{\Phi} \\
+ \epsilon_{MPS,t}
\]

where \( \epsilon_{MPS,t} \) is a monetary policy shock and

\[
\varphi E_t[\log \Theta_{t+1}] = \log \Theta_t - (1 - \varphi) \log(\Pi_t)
\]

\[
\log \Phi_t = \log \Pi_t + \tau \log \Phi_{t-1}
\]

- Interpret as a feedback from expected inflation with mean forecast horizon \((1 - \varphi) \sum_{h=1}^{\infty} h \varphi^h = \varphi / (1 - \varphi)\)

- Similarly, \( \tau \) can be interpreted as the degree of backward-lookingness of the monetary authority
### Results for NK Model: US

<table>
<thead>
<tr>
<th>Model</th>
<th>Standard Deviation</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Model</td>
<td>Output</td>
<td>Inflation</td>
<td>Interest rate</td>
<td>Investment</td>
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<tr>
<td>Data</td>
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<table>
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<table>
<thead>
<tr>
<th>Autocorrelations (Order=1)</th>
</tr>
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</table>

**Table:** Selected Second Moments - US Economy (80:1-06:4)
## Results for NK Model: India

<table>
<thead>
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<td>Inflation</td>
<td>Interest rate</td>
<td>Investment</td>
<td></td>
</tr>
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<td>Data</td>
<td>1.22</td>
<td>0.97</td>
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### Cross-correlation with Output

<p>| | | | | | |</p>
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<tbody>
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<td>0.05</td>
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### Autocorrelations (Order=1)

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<table>
<thead>
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</thead>
<tbody>
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<td>Data</td>
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<td>0.71</td>
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*Table: Selected Second Moments - Indian Economy (96:1-08:4)*
Financial Frictions: The Financial Accelerator

- Financial Accelerator facing firms: risk premium $\uparrow$ with leverage

\[
\begin{align*}
\text{Balance Sheet:} & \quad Q_t \cdot K_t = N_t + B_t \\
\text{Capital} & \quad \text{Equity} \quad \text{External Finance} \\
\text{Leverage:} & \quad \frac{B_t}{N_t} = \frac{Q_t \cdot K_t - N_t}{N_t} \\
\text{Risk Premium} & \quad \Theta_t = k \left( \frac{Q_t \cdot K_t}{N_t} \right)^{\chi} RPS_t \\
\end{align*}
\]

- Suppose $\Theta$ is observed (see [Haugen(2005)].) Let $n_k \equiv \frac{N}{Q} = \frac{1}{1+\ell}$ where $\ell$ is leverage. Then we can set the scaling parameter $k$ as $k = \Theta n_k^\chi$
Financial Frictions: Rule of Thumb Consumers

- A proportion $\lambda$ of households are credit-constrained

$$C_t = \lambda C_{1,t} + (1 - \lambda) C_{2,t}$$

Non-Ricardian Ricardian

$$C_{1,t} = \frac{W_t h_t}{P_t} = \text{Wage Income}$$

- $C_{2,t}$ given by the standard Euler-consumption equation
The Informal Sector

According to [Sen and Kolli(2009)] and [Rada(2009)] the broad characteristics of the informal sector are

- Individual or household enterprises
- No complete accounts
- Produces some marketable goods and services
- Not registered
- 90% workers are in the I-sector producing 50% of GDP

<table>
<thead>
<tr>
<th></th>
<th>Lab. Market</th>
<th>Credit Market</th>
<th>Taxation</th>
<th>Lab. Share</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>F Sector</strong></td>
<td>frictions</td>
<td>lower frictions</td>
<td>taxed</td>
<td>lower</td>
</tr>
<tr>
<td><strong>I Sector</strong></td>
<td>no frictions</td>
<td>higher frictions</td>
<td>untaxed</td>
<td>higher</td>
</tr>
</tbody>
</table>
The Model: Overview

- We consider a two-sector “Formal” (F) and “Informal” (I) economy, producing different range of differentiated goods with different technologies which sell at different aggregate retail prices, $P_{F,t}$ and $P_{I,t}$
- Distortionary (employment) taxes in the F-sector
- Real wage norm in the F-sector
- Capital and government services part of the F-sector
- Different FAs in the two sectors; higher steady-state risk premium in the I-sector
- A proportion $n_{F,t}$ of Ricardian households work in the F-sector. All non-Ricardian households work in the I-sector
The Model: Structure

[Diagram showing the structure of the model with sectors and flows labeled: Entrepreneurs, Retail Firms (I), Retail Firms (F), Wholesale Firms (I), Wholesale Firms (F), Government, Capital Producers, Labour Market (I), Labour Market (F), Ricardian Households, Non-Ricardian Households, and Formal Sector, Informal Sector.]
The Model: Some Details

- The real wage in the F-sector given by a real wage norm \( RW_t \) that is a mark-up \( rw \) on the real wage in the informal sector:

\[
\frac{W_{F,t}}{P_t} = RW_t = (1 + rw) \frac{W_{I,t}}{P_t}; \quad rw > 0
\]

- On the demand side of the model we construct Dixit-Stiglitz consumption and price aggregates

\[
C_t = \left[ \frac{1}{w} C_{F,t}^{\frac{\mu-1}{\mu}} + (1 - w) C_{I,t}^{\frac{\mu-1}{\mu}} \right]^{\frac{\mu}{\mu-1}}
\]

\[
P_t = \left[ w(P_{F,t})^{1-\mu} + (1 - w)(P_{I,t})^{1-\mu} \right]^{\frac{1}{1-\mu}}
\]

where \( \mu \) is the elasticity of substitution between I and F goods, and \( w \) is a preference “parameter”. Then standard results are:

\[
C_{F,t} = w \left( \frac{P_{F,t}}{P_t} \right)^{-\mu} C_t; \quad C_{I,t} = (1 - w) \left( \frac{P_{I,t}}{P_t} \right)^{-\mu} C_t
\]
Calibration and Priors

- We calibrate the model to fit two variables for which we have information: relative nos of workers ($rel_n$) and relative GDP contributions ($rel_Y \equiv \frac{P_F Y_F}{P_I Y_I}$). Let $n_F$ be the proportion of Ricardian households in the F-sector. From the model we have:

$$rel_n = \frac{(1 - \lambda)n_F}{(1 - \lambda)(1 - n_F) + \lambda} \quad (1)$$

$$rel_Y = \frac{P_F Y_F}{P_I Y_I} = \frac{w \left( \frac{P_F}{P} \right)^{1-\mu} \bar{C}_t + \left( \frac{P_F}{P} \right)(\bar{I}_t + \bar{G}_t)}{(1 - w) \left( \frac{P_I}{P} \right)^{1-\mu} \bar{C}_t} \quad (2)$$

- From (2) we can solve for $w$ to obtain

$$w = \frac{rel_Y \left( \frac{P_I}{P} \right)^{1-\mu} \bar{C}_t - \left( \frac{P_F}{P} \right)(\bar{I}_t + \bar{G}_t)}{\left( \frac{P_F}{P} \right)^{1-\mu} \bar{C}_t + rel_Y \left( \frac{P_I}{P} \right)^{1-\mu} \bar{C}_t}$$
Calibration of \( rw \) and \( \alpha_I \) to fit \( rel_n \) and \( rel_Y \).

Labour shares \( \alpha_I = 0.8 > \alpha_F \).
### Bayesian Estimation

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Notation</th>
<th>Prior distribution</th>
<th>Posterior distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calvo prices (F)</td>
<td>$\xi_F$</td>
<td>Beta</td>
<td>0.75 0.15</td>
</tr>
<tr>
<td>Calvo prices (I)</td>
<td>$\xi_I$</td>
<td>Beta</td>
<td>0.75 0.15</td>
</tr>
<tr>
<td>Labour share (F)</td>
<td>$\alpha_F$</td>
<td>Beta</td>
<td>0.60 0.10</td>
</tr>
<tr>
<td>Labour share (I)</td>
<td>$\alpha_I$</td>
<td>Beta</td>
<td>0.80 0.10</td>
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<tr>
<td>Preference parameter</td>
<td>$\varphi$</td>
<td>Beta</td>
<td>0.50 0.20</td>
</tr>
<tr>
<td>Degree of Labour market frictions</td>
<td>$r_W$</td>
<td>Beta</td>
<td>0.75 0.10</td>
</tr>
<tr>
<td>GDP Contribution</td>
<td>$rel_Y$</td>
<td>Normal</td>
<td>1.00 0.50</td>
</tr>
<tr>
<td>Employment Ratio</td>
<td>$reln$</td>
<td>n.a.</td>
<td>0.20 n.a.</td>
</tr>
<tr>
<td>External finance premium elasticity (F)</td>
<td>$\chi_F$</td>
<td>Inv. gamma</td>
<td>0.05 4.00</td>
</tr>
<tr>
<td>External finance premium elasticity (I)</td>
<td>$\chi_I$</td>
<td>Inv. gamma</td>
<td>0.05 4.00</td>
</tr>
<tr>
<td>Inverse of Leverage (F)</td>
<td>$n_F$</td>
<td>Beta</td>
<td>0.5 0.15</td>
</tr>
<tr>
<td>Inverse of Leverage (I)</td>
<td>$n_I$</td>
<td>Beta</td>
<td>0.5 0.15</td>
</tr>
<tr>
<td>Proportion of RT consumers</td>
<td>$\lambda$</td>
<td>Beta</td>
<td>0.40 0.10</td>
</tr>
<tr>
<td>Interest rate rule</td>
<td></td>
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<td></td>
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<tr>
<td>Interest rate smoothing</td>
<td>$\rho$</td>
<td>Beta</td>
<td>0.75 0.10</td>
</tr>
<tr>
<td>Feedback from expected inflation</td>
<td>$\theta$</td>
<td>Normal</td>
<td>2.00 1.00</td>
</tr>
<tr>
<td>Feedback from past inflation</td>
<td>$\phi$</td>
<td>Normal</td>
<td>2.00 1.00</td>
</tr>
<tr>
<td>Degree of forward-lookingness</td>
<td>$\varphi$</td>
<td>Beta</td>
<td>0.50 0.20</td>
</tr>
<tr>
<td>Degree of backward-lookingness</td>
<td>$\tau$</td>
<td>Beta</td>
<td>0.50 0.20</td>
</tr>
</tbody>
</table>

◊ We report posterior means and 95% probability intervals (in parentheses) except for $reln$ which has an unknown distribution.
Bayesian Estimation: Summary

- Model Comparison: Decisive Support for FF and an I-Sector

<table>
<thead>
<tr>
<th>Model</th>
<th>LL</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>NK</td>
<td>-358.95</td>
<td>0.0005</td>
</tr>
<tr>
<td>NK+FF</td>
<td>-356.28</td>
<td>0.0067</td>
</tr>
<tr>
<td>NK+FF+I-Sector</td>
<td>-350.73</td>
<td>0.9928</td>
</tr>
</tbody>
</table>

- Average Price Contract lengths: 4.02 quarters (F-sector) and 1.43 (I-sector). Suggests I-sector prices are more flexible
- I-sector less leveraged, but with a weaker FA
- Stronger forward-looking response to inflation in the Calvo interest rate rule; but data is not informative on average lags
- Data is informative about the size of the informal sector.
## Model Validation I: Selected Second Moments

<table>
<thead>
<tr>
<th>Model</th>
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<td>1.41</td>
</tr>
<tr>
<td>NK Model with FA</td>
<td>2.42</td>
<td>1.02</td>
<td>1.76</td>
</tr>
<tr>
<td>2-sector NK Model</td>
<td><strong>1.59</strong></td>
<td><strong>0.97</strong></td>
<td><strong>1.84</strong></td>
</tr>
</tbody>
</table>

**Table:** Selected Second Moments - Indian Economy (96:1-08:4)
Model Validation II: Autocorrelations

![Graphs showing autocorrelations for Output, Inflation, Interest rate, and Investment for different models: NK Model, NK Model with FA, 2-sector NK Model.](image)
Work-in-progress

- **Which Steady-State?** We have assumed a non-growth zero-inflation steady state. We should be assuming a balanced-growth non-zero-inflation steady state with a stochastic trend as in [Schmitt-Grohe and Uribe (2008)]

- Trending issues? Trend agnostic one-step estimation (Ferroni, 2010)

- Endogenous Calvo Price Contracts that decrease in duration as steady-state inflation increases

- Add openness and do the policy analysis to complete the project
  - Model_SOE 1: A standard open-economy NK model
  - Model_SOE 2: An open-economy NK model with financial frictions
  - Model_SOE 3: An open-economy NK model with financial frictions and incomplete exchange rate pass-through.


A Floating versus Managed Exchange Rate Regime in a DSGE Model of India.
University of Surrey Working Paper.


The Informal Credit Market: A Study of Default and Informal Lending in Nepal.
Mimeo, University of Bergen.

