New Eurocoin: Tracking Economic Growth in Real Time *

Filippo Altissimo[1], Riccardo Cristadoro[2] ,

Mario Forni[3], Marco Lippi[4], and Giovanni Veronese[2]

Abstract

Removal of short-run dynamics from a stationary time series to isolate the medium to long-run component can be obtained by a band-pass filter. However, band pass filters are infinite moving averages and can therefore deteriorate at the end of the sample. This is a well-known result in the literature isolating the business cycle in integrated series. We show that the same problem arises with our application to stationary time series. In this paper we develop a method to obtain smoothing of a stationary time series by using only contemporaneous values of a large dataset, so that no end-of-sample deterioration occurs. Our method is

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applied to the construction of New Eurocoin, an indicator of economic activity for the euro area, which is an estimate, in real time, of the medium to long-run component of the GDP growth. As our dataset is monthly and most of the series are updated with a short delay, we are able to produce a monthly, real-time indicator. As an estimate of the medium to long-run GDP growth, Eurocoin performs better than the band-pass filter *at the end of the sample*, both in terms of fitting and turning-point signaling.

Keywords: coincident indicator, band-pass filter, large-dataset factor models, generalized principal components.

JEL classification: C51; E32; O30.

[1]Brevan Howard Asset Management. [2]Bank of Italy. [3]Università di Modena e Reggio Emilia, CEPR and RECent. [4]Università di Roma 'La Sapienza'. Correspondence: ml@lippi.ws

1 Introduction

This paper presents a method to estimate in real time the current state of the economy, with an application to the euro area. The resulting indicator, New Eurocoin (NE henceforth), is intended to replace the Eurocoin indicator proposed by Altissimo et al. (2001) and published monthly by the Centre for Economic Policy Research (see the website www.cepr.org).

The main objective of NE is to make an assessment of economic activity that is (a) comprehensive and non-subjective, (b) timely and (c) free from short-run fluctuations.

Requirements (a) is obvious. Regarding (b) and (c), both private agents and economic policy makers require for their decisions a clear distinction, in real time, between transitory and long-lasting changes in the state of the economy. For example, if an upward change occurs it is crucial to decide whether it is the beginning of a long positive swing or a short-lived phenomenon. In particular, a counter-cyclical policy should target medium rather than short-term waves. The latter are both less detrimental and more difficult to fight, owing to the delays of policy reactions and of the effects of intervention on economic activity.

None of the available macroeconomic series provides a measure of the state of the economy that fulfills criteria (a), (b) and (c). The Gross Domestic Product (GDP), the most comprehensive indicator of real activity, fails to meet (b) and (c). Regarding timeliness, the GDP is only available quarterly and with a long delay. For instance, the preliminary estimate of euro area GDP for the first quarter of the year becomes available only in May. Moreover, the GDP is affected by a sizeable short-run component.

NE is a real-time estimate of GDP growth, cleaned of short-run oscillations. More precisely:

(i) We focus on the growth rate of the GDP and define the *medium to long-run growth*, henceforth denoted by MLRG, as the component of the GDP growth rate obtained by removing the fluctuations of period shorter than or equal to one year. This component, which is of course a smoothing of the GDP growth, is our ideal target.

(ii) NE is a monthly and timely estimate of the MLRG for the euro area: around the 20-th of each month we are able to produce a reliable estimate for the previous month.

To avoid possible misunderstandings, let us point out that "medium to long-run growth" only denotes the smoothed component of the growth rate defined above, and bears no relationship to any definition of trend. In particular, integration of the medium to long-run component will never be considered.

The MLRG, as defined above, is obtained by applying a band-pass filter. The latter, however, is an infinite, two-sided, moving average. Empirical applications imply replacing missing with predicted data, and therefore a possible deterioration at the end of the sample. In particular, poor end-of-sample estimation and serious revisions as new data become available have been consistently stated in the literature trying to isolate business cycle fluctuations in macroeconomic integrated time series (see Baxter and King, 1999, pp. 579-80, Christiano and Fitzgerald, 2003, pp. 459-60). The same end-of-sample problem arises applying band-pass filters to any stationary time series. However, the present paper concentrates on a particular stationary series, the euro-area GDP growth, and the band-pass filter that removes one-year or shorter fluctuations. The end-of-sample deterioration, for this case, is discussed in the paper and assessed in a real-time exercise in Section 6.

A substantial mitigation of this conflict between timeliness and removal of the short-run fluctuations is the main contribution of the present paper. Our indicator NE, which is an estimate of the MLRG, is based on a large dataset, including 145 euro-area macroeconomic variables. We construct a small number of "smooth factors", which are generalized principal components of *current* values of the variables in the dataset, specifically designed to remove short-run and variable-specific sources of fluctuation. NE is obtained as a linear combination of the smooth factors. As only current values of the variables are used, no end-of-sample deterioration occurs. Moreover, although NE cannot compete with the truncated band-pass filter within the sample, we show that NE outperforms the band-pass filter at the end of the sample, both in terms of fitting and turning-point signaling.

This result can be explained by observing that (i) the dataset contains variables that are leading with respect to current GDP, (ii) the smoothness of our factors is obtained by linearly combining current values of variables that are lagging, coincident and leading with respect to the GDP. Therefore the information contained in future values of the GDP, which are unavailable at the end of the sample, can be partially recovered using the smooth factors.

The method used in the present paper is based on the large-scale Generalized Dynamic Factor Model (GDFM) proposed in Forni, Hallin, Lippi and Reichlin (2000, 2005), Forni and Lippi (2001) (see also the literature cited in Section 5). Valle e Azevedo, Koopman and Rua (2006) propose a multivariate method with band-pass filter properties which exploits information from a relatively small number of variables. We are not far in spirit from their work, the main difference being that our procedure is designed to extract information from a large panel of time series.

Let us point out that our ideal target, being an infinite moving average, is, strictly speaking, unobservable. However, as we show in Appendix A, our finite-sample version of the band-pass filter provides a good approximation to the ideal target until we are one year away from the end and beginning of the sample. This is the basis for our adopted empirical target and measure of performance for NE. Precisely, the performance of NE at time t, with $t \leq T - 12$, is measured as the difference between NE at time t, and the empirical target at t, obtained using the data up to T.

The paper is organized as follows. Section 2 collects some preliminary observations. Section 3 defines our target, i.e. the medium to long-run component of GDP, and discusses its interpretation. Sections 4 and 5 describe and motivate our estimation procedure. Section 6 constructs the New Eurocoin indicator and analyzes its real-time performance in comparison with alternative indicators. Section 7 concludes. The Appendix contains a detailed discussion of the ideal target, the empirical target and their distance, a description of the dataset and a short comparison between New and Old Eurocoin.

2 Preliminary observations

To gauge the current state of the economy, given the delay with which GDP is released, market analysts and forecasters resort to more timely and higher frequency information and on this basis obtain early estimates of GDP. However, two problems immediately arise: (i) looking at the typical release calendar for the euro area, one can see that timeliness varies greatly even among monthly statistics (end-of-sample unbalance); (ii) since GDP is quarterly we have to handle simultaneously monthly and quarterly data.

In what follows we show how to combine the comprehensive and non-subjective information provided by GDP with the early information provided by surveys and other monthly series to obtain a reliable and timely picture of current economic activity.

Our dataset includes monthly series of consumer and production prices, wages, share prices, money, unemployment rates, job vacancies, interest rates, exchange rates, industrial production, orders, retail sales, imports, exports, and consumer and business surveys for the euro area countries and the euro area as a whole (see Appendix B for details). The dataset has been organized taking into account the calendar of data releases that is typical in real situations, with the aim of reproducing the staggered flow of information available through time to policy-makers and market forecasters. This lack of synchronism, though little considered in the literature, is crucial for assessing realistically the performance of alternative real-time indicators.[1]

Table 1: **The calendar of some macroeconomic series**

Time / *Release date*	DEC. 04	GEN. 05	FEB. 05	MAR. 05	APR. 05	MAY 05	JUN. 05		
	Real time information sets →								***Delay***
GDP	**Q3 - 2004**	**Q3 - 2004**	**Q4 - 2004**	**Q4 - 2004**	**Q4 - 2004**	**Q1-2005**	**Q1-2005**		*45-90 days*
Industrial production	Oct. 04	Nov. 04	Dec. 04	Jan. 05	Feb. 05	Mar. 05	Apr. 05		*45-50 days*
Surveys	Dec. 04	Jan. 05	Feb. 05	Mar. 05	Apr. 05	May. 05	Jun. 05		*0-25 days*
Retail sales	Oct. 04	Nov. 04	Dec. 04	Jan. 05	Feb. 05	Mar. 05	Apr. 05		*45-50 days*
Financial markets	Dec. 04	Jan. 05	Feb. 05	Mar. 05	Apr. 05	May. 05	Jun. 05		*0 days*
CPI	Nov. 04	Dec. 04	Jan. 05	Feb. 05	Mar. 05	Apr. 05	May. 05		*15 days*
Car registrations	Nov. 04	Dec. 04	Jan. 05	Feb. 05	Mar. 05	Apr. 05	May. 05		*2-30 days*
Industrial orders	Oct. 04	Nov. 04	Dec. 04	Jan. 05	Feb. 05	Mar. 05	Apr. 05		*50 days*

[1]Important exceptions are Bernanke and Boivin (2003) and Giannone et. al. (2002).

As illustrated in Table 1, Financial Variables and Surveys are the most timely data, while Industrial Production and other "real variables" are usually available with longer delays. Around the 20-th of month $T + 1$, when we calculate the indicator for month T, Surveys and Financial Variables are usually available up to time T, thus with no delay, Car Registrations and Industrial Orders up to $T - 1$ and Industrial Production indexes up to $T - 2$ or $T - 3$. The GDP series is observed quarterly, so that its delay varies with time. For example, on the 20-th of April only data up to the fourth quarter of the previous year are available, thus a three-month delay with respect to T, which is March. At the 20-th of May the delay with respect to T is reduced to one month, as a first-quarter preliminary estimate is released, and will be two months when T is May, hence an average delay of two months.

The most timely variables (such as Purchasing Managers Indexes, Consumer Surveys, Business Climate Indexes, etc.) are far from being comprehensive and smooth. Other standard series, such as Industrial Production and Exports, complement the information content of the surveys but are less timely. Furthermore, all monthly series exhibit heavy short-run fluctuations and might provide contradictory signals, see Figure 1. As a result, none of them is fully satisfactory and "there is much diversity and uncertainty about which indicators are to be used" (Zarnowitz and Ozyildirim, 2002).

We tackle the end-of-sample unbalance in the following way. Let x^*_{it}, $i = 1, \ldots, n$, be the series after outliers and seasonality have been removed and stationarity achieved by a suitable transformation (see Appendix B). Let k_i be the delivery delay (in months) for variable x^*_{it}, so that when we are at the end of the sample its last available observation is

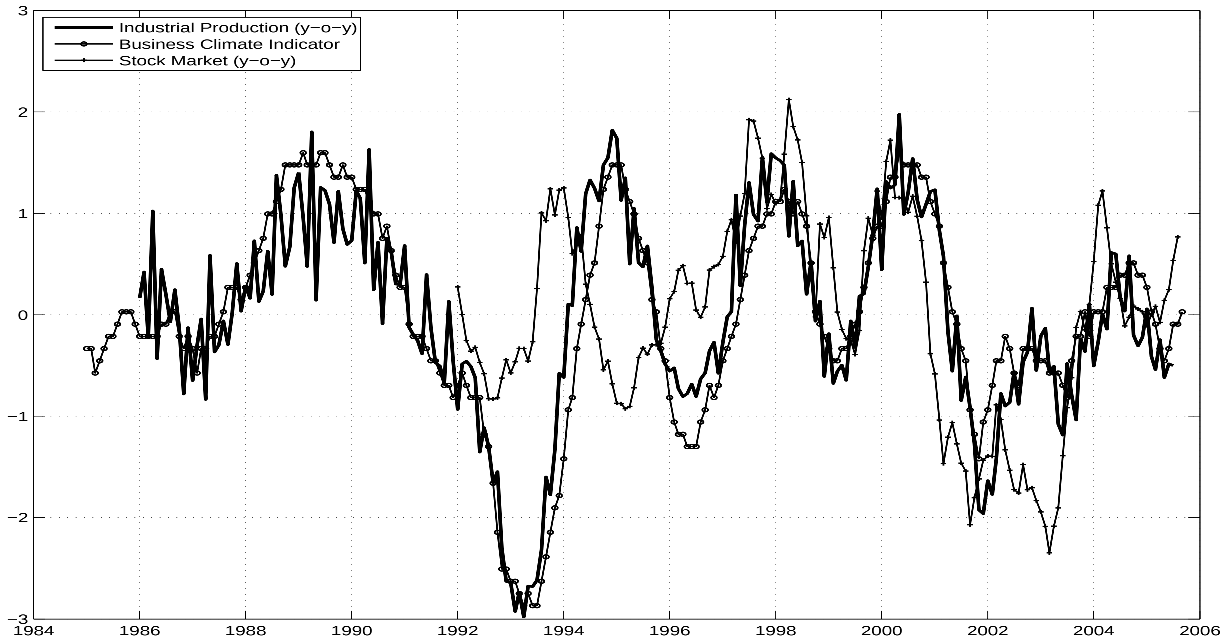

Figure 1: **Some economic indicators for the euro area (normalized scale)**

$x^*_{i,T-k_i}$. We define the panel x_{it}, $i = 1, \ldots, n$, by setting

$$x_{it} = x^*_{i,t-k_i}, \tag{1}$$

so that the last available observation of x_{it} is at T for all i. Of course, this realignment implies cutting some observations at the beginning of the sample for several variables. As a result, after transforming and realigning, the dataset goes from June 1987 to June 2005, hence $T = 217$. The same realignment is used both when we consider the whole sample, up to T, and when we consider subsamples $[1\ \tau]$, as in the pseudo real-time exercises carried out in Section 6.

To use our monthly dataset to obtain a timely GDP indicator it is convenient to think of GDP as a *monthly series of quarterly aggregates with missing observations*. The figure for month t, denoted by z_t, is defined as the aggregate of GDP for months t, $t-1$ and $t-2$, so that there is a two-month overlapping between two subsequent elements of the series. Obviously, the monthly series is observable only for March, June, September and December.

The monthly GDP growth rate is defined as

$$y_t = \log z_t - \log z_{t-3}.$$

Thus y_t is the usual quarter-on-quarter growth rate, except that it is defined for all months.

How to deal with the missing observations in GDP will be discussed in detail in Section 3 and in Appendix A.1.

3 The MLRG and its interpretation

A natural way to define the medium to long-run fluctuations of a time series is by considering its *spectral representation*. Assuming stationarity, y_t can be represented as an integral of sine and cosine waves with frequency ranging between $-\pi$ and π, with respect to a stochastic measure (see e.g. Brockwell and Davis, 1991, ch. 4). Based on the spectral representation, we define the medium to long-run component of y_t by taking the integral over the interval $[-\pi/6\ \pi/6]$ instead of $[-\pi\ \pi]$. The frequency $\pi/6$ for a monthly series corresponds to a one-year period, thus we cut off seasonal and other higher frequency waves.

This frequency-domain construction has a time-domain counterpart, which is known as band-pass filter. Here we do not delve into the details of this correspondence and go directly to the result (see e.g. Baxter and King, 1999, and Christiano and Fitzgerald, 2003). Our medium to long-run component, call it c_t, is the following infinite, symmetric, two-sided linear combination of the GDP growth series:

$$c_t = \beta(L)y_t = \sum_{k=-\infty}^{\infty} \beta_k y_{t-k}, \quad \beta_k = \begin{cases} \dfrac{\sin(k\pi/6)}{k\pi} \quad \text{for} \quad k \neq 0 \\ 1/6 \quad \text{for} \quad k = 0. \end{cases} \tag{2}$$

The time series y_t has therefore the decomposition

$$y_t = c_t + s_t = \beta(L)y_t + [1 - \beta(L)]y_t, \tag{3}$$

where s_t includes all the waves of period shorter than one year. Since $\beta(1) = 1$, the mean of the GDP growth series, denoted by μ, is retained in c_t while the mean of s_t is zero. Because c_t and s_t are orthogonal, the variance of y_t is broken down into the sum of a short-run variance and a medium to long-run variance. The medium to long-run component c_t is our ideal target MLRG.

Application of (2) to the GDP growth rate requires some elaboration:

(i) Firstly, as we know, y_t is not observed monthly. Several solutions are possible, including linear interpolation of the missing values or the more sophisticated techniques introduced in Chow and Lin (1971). However, as far as the variable c_t is concerned, the particular interpolation of the missing values in y_t makes no significant difference (see Appendix A.1 for details).

(ii) Thus we choose linear interpolation. Precisely, consider the *months* from 1 to $\tau \leq T$. We assume that 1 is the first publication date of the GDP and denote by T_P the last publication date within $[1\ \tau]$. Moreover, denote by $\hat{\mu}$ the mean of y_t, estimated using its quarterly observations within $[1\ \tau]$. Then define $\check{y}_t$ by setting $\check{y}_t = y_t$ for $t = 1,\ 4,\ \ldots,\ T_P$

and

$$y_t = \hat{\mu} \text{ for } t = -2,\ -5,\ -7, \ldots \text{ and } t = T_P + 3,\ T_P + 6,\ T_P + 9.\ \ldots$$

The series $\check{y}_t$ is infinite, with two missing observations for each quarter.

(iii) Call $y_t(\tau)$ the result of the linear interpolation of $\check{y}_t(\tau)$ and define, for $t = 1, 2, \ldots, \tau$,

$$c_t^*(\tau) = \beta(L) y_t(\tau). \tag{4}$$

Thus we use the notation $c_t^*(T)$ when the whole interval $[1\ T]$ is considered, or simply c_t^* if no confusion arises.

In Appendix A we show that $c_t^*(T)$ provides a very good approximation of c_t for $13 \leq t \leq T - 12$, where $T = 217$, the size of our sample. Our argument is based both on a simulation exercise and theoretical calculations.

The simulation design is as follows. Mimicking the dynamic structure of y_t, we generate 2000 time series of length $M = 2N + T + 2N$:

$$y_{j,t}, \quad j = 1, \ldots, 2000; \quad t = -2N + 1, \ldots, 0, \ \underbrace{1, \ldots, T,}\ \ T + 1, \ldots, T + 2N.$$

A preliminary simulation determines N as such that the revision

$$c_{j,t}^*(2N + T + 2N) - c_{j,t}^*(N + T + N),$$

is negligible for all $1 \leq t \leq T$ and all $j = 1, \ldots, 2000$ (see Appendix A.2). As a consequence, setting $\hat{M} = N + T + N$, for t belonging to the central subsample of length T, we set $c_{j,t} = c_{j,t}^*(\hat{M})$. Then for $1 \leq q \leq T$ we consider the ratio

$$v_{j,t,T} = \frac{(c_{j,t}^*(T) - c_{j,t})^2}{\sigma_j^2},$$

where σ_j^2 is the estimated variance of $c^*_{j,t}(\hat{M})$. Denoting by $V_{t,T}$ its average over 2000 replications, we find, for $T = 217$,

$$V_{T,T} = 0.14, \quad V_{T-12,T} = 0.008, \quad V_{T-108,T} = 0.0013,$$

thus a close approximation up to $T - 12$, followed by a rapid deterioration (further details on the deterioration in Appendix A.2).

A similar pattern, as shown in Appendix A.3, results from the theoretical frequency-domain calculation of the ratio

$$\frac{\text{var}(c^*_t(T) - c_t)}{\text{var}(c_t)},$$

where c_t and $c^*_t(T)$ are obtained by applying, respectively, $\beta(L)$ and the truncated version equivalent to (4) (see Appendix A.3), to several monthly stationary processes.

We henceforth take $c^*_t(T)$ as our *empirical target* for $13 \leq t \leq T - 12$. In Section 6 we use $c^*_t(T)$, within $[13 \quad T - 12]$, to compare the performance of NE and other indicators, both in terms of fit and ability to signal turning points.

Figure 2 presents the approximation $c^*_t(T)$ for the euro zone GDP, $13 \leq t \leq T - 12$, along with quarterly GDP growth, y_t, where T is June 2005. We see that c^*_t closely tracks the GDP growth (it captures about 70% of the variance of y_t).

Figure 2 provides a clear illustration of the smoothing effect of the band-pass filter. Short-run waves are removed, so that observers can distinguish longer oscillations and their turning points. The main task of the paper is a good estimate of c_t at the end of the sample, so that turning points can be detected in real time (see Section 6).

We conclude this section with a few observations about the relationship between MLRG and the year-on-year change of GDP, which is often reported as a measure of

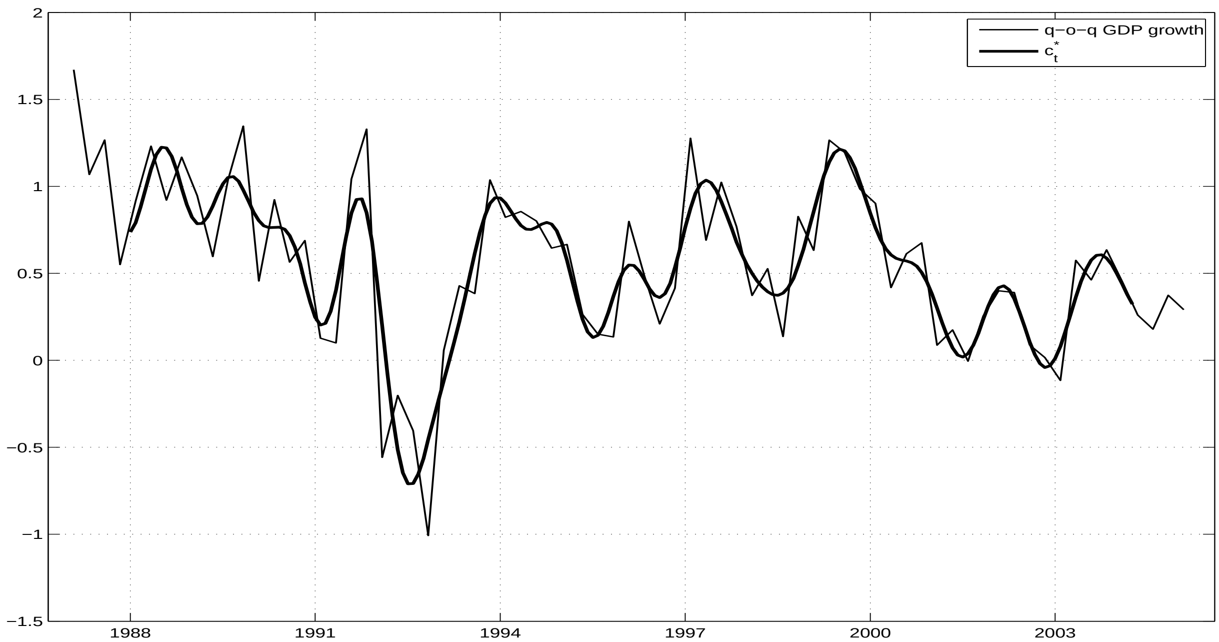

Figure 2: $c_t^*(T)$ **and the monthly quarter-on-quarter GDP growth rate**

medium to long-run growth. Indicating by $\tilde{y}_t$ the year-on-year change of GDP, i.e. the difference between the quarter ending at t and the quarter ending at $t-12$ (divided by 4 to obtain quarterly rates) we have

$$\tilde{y}_t = \frac{y_t + y_{t-3} + y_{t-6} + y_{t-9}}{4}.$$

Hence $\tilde{y}_t$ is a moving average of the y series which, unlike MLRG, is one-sided towards the past and hence not centered at t. As a result, $\tilde{y}_t$ is lagging with respect to both y_t and MLRG by several months (precisely four and a half), as is apparent from Figure 3.

The phase shift is reduced if we compare MLRG with the future of $\tilde{y}_t$. In Section 6.4 we show that our indicator, which tracks MLRG, is a good predictor of future year-on-year growth.

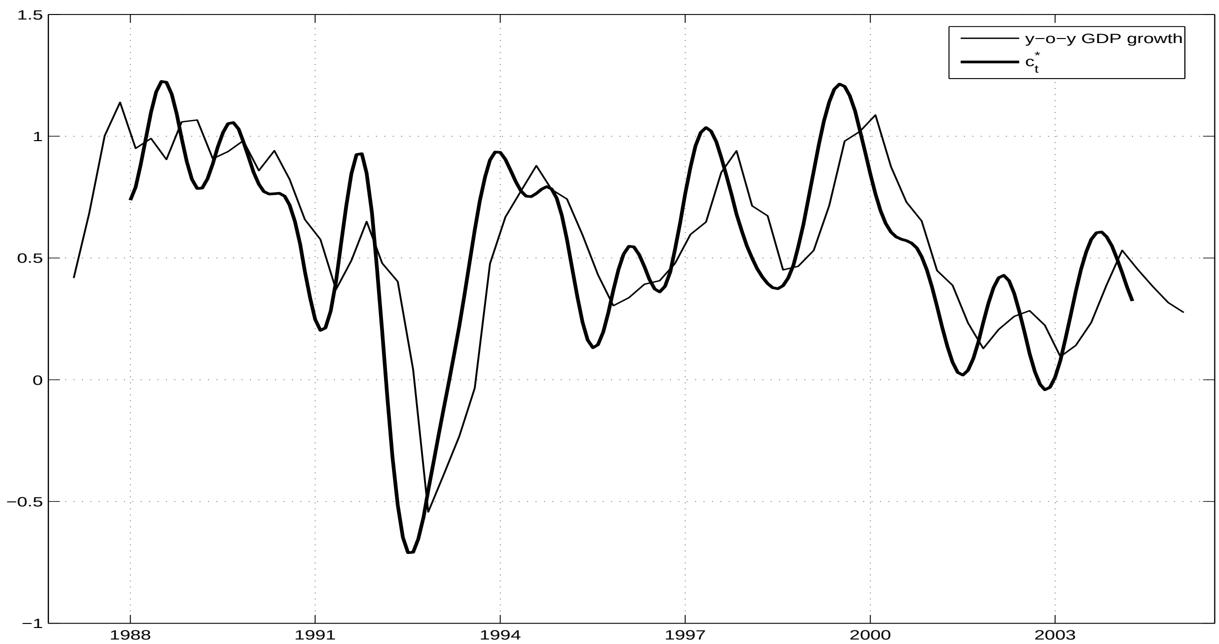

Figure 3: $c_t^*(T)$ **and the monthly year-on-year GDP growth rate**

4 Estimation I: projecting the MLRG on monthly regressors

We now begin the construction of NE, our alternative estimate of c_t. A brief outline of our procedure will be helpful to the reader:

(i) NE is the projection of c_t on a set of regressors, which are linear combinations of the variables in the dataset. In the present section we give a detailed description of the way we compute the projection once the regressors have been constructed.

(ii) In Section 5 we construct the regressors. Assuming that our dataset can be modeled as a dynamic factor model, we determine the dimension of the factor space, call it r, and project c_t on the first r principal components of the series in the dataset, which is a basis of the factor space, call κ_t the projection. Our regressors are generalized principal components, specifically designed to minimize the short-run component. For this reason we call them smooth factors. In Section 6 we determine the number of smooth factors as the integer $\bar{s}$ such that the residual of the projection κ_t and that of the projection of c_t on the first $\bar{s}$ smooth factors are approximately of the same size. We show that $\bar{s}$ is smaller than r and that the projection on the $\bar{s}$ smooth factors is substantially smoother than κ_t. (iii) This projection, of c_t on the first $\bar{s}$ smooth factors is NE. In Section 6 we provide a detailed assessment of the real-time performance of NE.

The variables in the dataset are observed monthly. The regressors, denoted by w_{kt}, $k = 1, \ldots, r$, are contemporaneous linear combinations of such variables and are therefore monthly variables. The projection of c_t on the regressors requires some discussion.

The population projection of c_t on the linear space spanned by $w_t = (w_{1t} \;\cdots\; w_{rt})'$ and the constant is

$$P(c_t|w_t) = \mu + \Sigma_{cw}\Sigma_w^{-1}w_t, \tag{5}$$

where Σ_{cw} is the row vector whose k-th entry is $\text{cov}(c_t, w_{kt})$ and Σ_w is the covariance matrix of w_t. NE is obtained by replacing the above population moments with estimators:

$$\hat{c}_t = \hat{\mu} + \hat{\Sigma}_{cw}\hat{\Sigma}_w^{-1}w_t. \tag{6}$$

Estimation of $\hat{\Sigma}_w$ is standard once the regressors w_t have been defined. Estimation of $\hat{\Sigma}_{cw}$ is less obvious:

(i) The covariances between c_t and w_t can be estimated using w_t and the approximation c_t^*, leaving aside end- and beginning-of-sample data.

(ii) Alternatively, we can start by estimating the cross-covariances between the quarterly series y_t and w_t. Note that this is possible for any *monthly* lead and lag[2], while it is not possible to estimate a monthly auto-covariance for y_t. Using such cross-covariances we obtain an estimate of the cross-spectrum between c_t and w_t, call it $\hat{S}_{cw}(\theta)$. Lastly, $\hat{\Sigma}_{cw}$ is obtained by integrating $\hat{S}_{cw}(\theta)$ over the band $[-\pi/6 \ \pi/6]$.

The results obtained with the two techniques do not differ substantially. The latter is more elegant[3] and has therefore been selected.

5 Estimation II: constructing the regressors

5.1 Dynamic factor models

The regressors w_{kt} are constructed using techniques from large-dimensional dynamic factor models. We assume that each of the variables x_{it} in the dataset is driven by a small number of common shocks, plus a variable-specific, usually called idiosyncratic, component. The idea that this common-idiosyncratic decomposition provides a useful description of macroeconomic variables goes back to the seminal work of Burns and Mitchell (1946) and has been recently developed in the literature on large-dimensional dynamic factor models; see Bai (2003), Bai and Ng (2002), Forni, Hallin, Lippi and Reichlin (2000, 2001, 2004, 2005; henceforth FHLR), Forni and Lippi (2001), and Stock and Watson (2002a, 2002b),

[2]Our method is closely related to the mixed-data sampling (MIDAS) approach, see Ghysels *et al.* (2007).

[3]We should keep in mind that the series $y_t(T)$, used construct $c_t^*(T)$ has been obtained by linear interpolation, so that $c_t^*(T)$, strictly speaking, it is not covariance stationary, nor costationary with the variables in the dataset.

Kapetanios and Marcellino (2004).

Large-dimensional factor models estimate a small (relative to the size of the dataset) number of "common factors", obtained as linear combinations of the x_{it}'s, which remove the idiosyncratic components and retain the common sources of variation. The innovation of the present paper with respect to this literature is a procedure to remove both the idiosyncratic and the short-run components, so that the resulting factors are both common and smooth.

Let us shortly recall the main features of the dynamic factor model we are referring to. Each series x_{it} in the dataset is the sum of a *common component*, call it χ_{it}, which is driven by a small number of common shocks, and an *idiosyncratic component*, ξ_{it}:

$$x_{it} = \chi_{it} + \xi_{it} = b_{i1}(L)u_{1t} + b_{i2}(L)u_{2t} + \cdots + b_{iq}(L)u_{qt} + \xi_{it}. \tag{7}$$

Common and idiosyncratic components are orthogonal at all lead and lags. Moreover, the idiosyncratic components ξ_{it} and ξ_{jt} are mutually orthogonal at all leads and lags for $i \neq j$.[4]

Model (7) is further specified by assuming that the common components χ_{it} can be given the static representation

$$\chi_{it} = c_{i1}F_{1t} + c_{i2}F_{2t} + \cdots + c_{ir}F_{rt}. \tag{8}$$

Under (8), different estimators, that are consistent as both the number of observations in each series (T) and the number of series in the dataset (n) tend to infinity, have been proposed for the space G_F spanned by the factors F_{jt}, see Stock and Watson (2002a, 2002b), and FHLR (2005). In particular, Stock and Watson use the first r principal

[4]This assumption can be relaxed. See the literature cited above.

components of the variables x_{it}. Consistent estimates of the common components χ_{it} are obtained by projecting the variables x_{it} on the estimated factors.

Our assumption is that the variables x_{it}, as well as the GDP are driven by the factors F_{kt}. On the other hand, y_t is a quarter-on-quarter rate of change, whereas the x's, i.e. the variables used to construct the factors, are month-on-month rates of change, so that, as we argue in Appendix A.4, representing y_t in terms of the factors transformed by $(1 + L + L^2)^2$, i.e.

$$y_t = c_{y1}[(1 + L + L^2)^2 F_{1t}] + c_{y2}[(1 + L + L^2)^2 F_{2t}] + \cdots + c_{yr}[(1 + L + L^2)^2 F_{rt}] + \xi_{yt},$$

is parsimonious and fairly reasonable. Thus the projection of c_t on the factor space will always be estimated by using the transformed regressors $(1 + L + L^2)^2 F_{kt}$, $k = 1, \ldots, r$ (the same transformation will be applied to the smooth factors, see Section 5.2).

Using our dataset over the whole sample period $[1\ T]$, the dimension of the factor space G_F has been estimated using the Bai-Ng criteria $\mathrm{PC}_{\mathrm{P1}}$ and $\mathrm{PC}_{\mathrm{P2}}$ (see Bai and Ng, 2002; we set $r_{\max} = 25$), the result being $r = 12$. Secondly, c_t has been projected on the first 12 principal components, filtered with $(1 + L + L^2)^2$, the projection being based on (6). This projection, denoted by κ_t, is shown in Figure 4 together with $c_t^*(T)$.[5]

We find that κ_t is a fairly good approximation to $c_t^*(T)$. Indeed the R^2 of the regression of $c_t^*(T)$ on κ_t, over the period $[13\ \ T - 12]$, is as high as 0.77. However, as Figure 4 shows, κ_t (upper panel, solid line) contains a sizable short-run component.

Smoother versions of κ_t can be obtained by reducing the number of principal components. As a matter of fact, the first principal component is quite smooth, but all the others, starting with the second, exhibit substantial short-run oscillations. The projec-

[5]We compute κ_t only for the whole sample. Therefore we do not need the notation $\kappa_t(T)$.

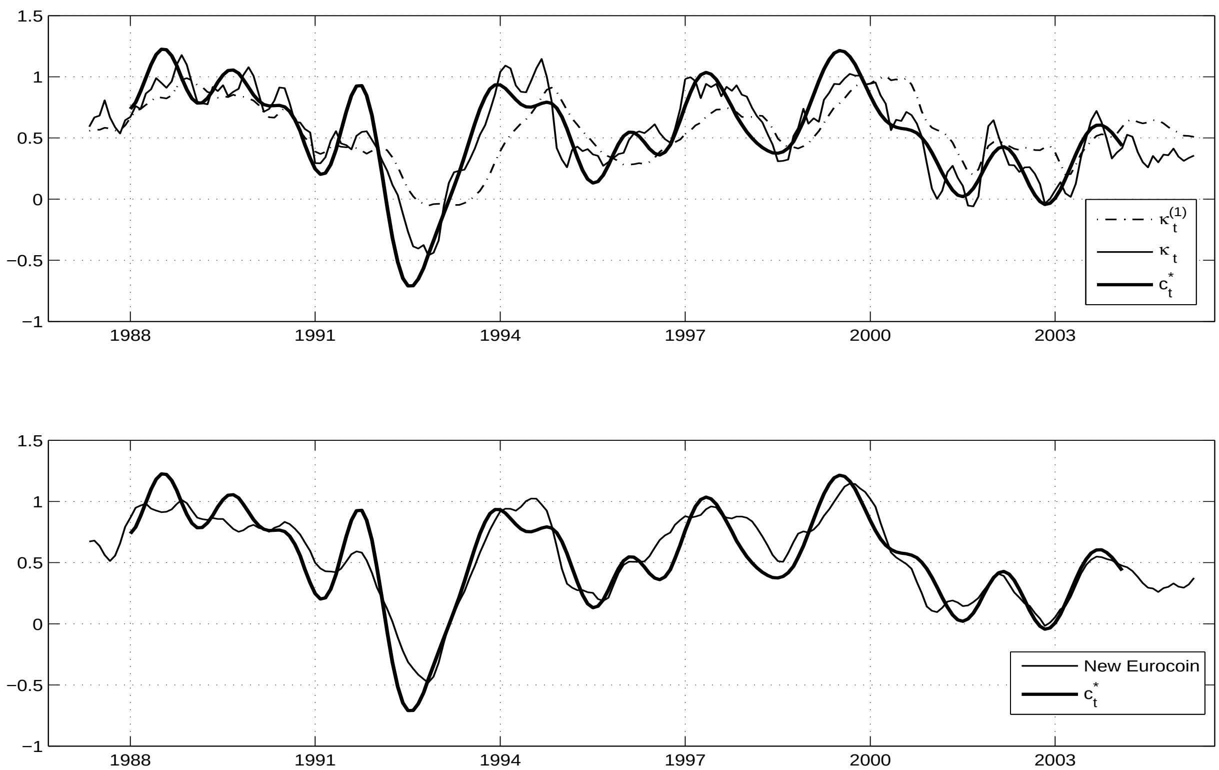

Figure 4: **Upper panel: $c_t^*(T)$, thick line, κ_t, solid line, $\kappa_t^{(1)}$, dash-dotted line. Lower panel: $c_t^*(T)$, thick line, New Eurocoin, solid line.**

tion of c_t on the first principal component (filtered with $(1 + L + L^2)^2$), call it $\kappa_t^{(1)}$, is plotted together with $c_t^*(T)$ in Figure 4 (upper panel, dash-dotted line). A considerable improvement in smoothness is obtained, but, firstly, the R^2 falls to 0.45, and, secondly, $\kappa_t^{(1)}$ has a systematic phase delay with respect to $c_t^*(T)$. As soon as we project on two principal components the gain in smoothness almost disappears.[6] The next subsection shows how smoothness can be obtained by a different definition of principal components.

[6]The plots of the projection of c_t on two, three, etc. principal components are available on request.

5.2 Smooth factors

We claim that by conveniently choosing a basis in G_F (different from the 12 principal components used to construct κ_t) we can obtain a projection with approximately the same fit but with a considerably reduced short-run component. Our construction is as follows. Let x_t, χ_t and ξ_t be the vectors of the variables x_{it}, their common components and their idiosyncratic components respectively. Let ϕ_{it} be the medium to long-run component of χ_{it}, precisely $\phi_{it} = \beta(L)\chi_{it}$, and $\psi_{it} = \chi_{it} - \phi_{it}$. For the spectral density matrices we have:

$$S_x(\theta) = S_\chi(\theta) + S_\xi(\theta) = S_\phi(\theta) + S_\psi(\theta) + S_\xi(\theta).$$

Integrating over the interval $[-\pi\ \pi]$, we obtain the following decompositions of the variance-covariance matrix of the x's:

$$\Sigma_x = \Sigma_\chi + \Sigma_\xi = \Sigma_\phi + \Sigma_\psi + \Sigma_\xi.$$

Consistent estimates $\hat{\Sigma}_\chi$, $\hat{\Sigma}_\phi$ and $\hat{\Sigma}_\xi$ can be obtained from the estimates of the spectral density $S_x(\theta)$. See Forni et al. (2000) for estimates of $S_\chi(\theta)$ and $S_\xi(\theta)$. $\hat{\Sigma}_\chi$ and $\hat{\Sigma}_\xi$ are obtained by integrating $\hat{S}_\chi(\theta)$ and $\hat{S}_\xi(\theta)$ over $[-\pi\ \pi]$, see Forni et al. (2005), $\hat{\Sigma}\phi$ by integrating $S_\chi(\theta)$ over $[-\pi/6\ \pi/6]$.

The matrices $\hat{\Sigma}_\chi$, $\hat{\Sigma}_\phi$ and $\hat{\Sigma}_\xi$ are all we need to construct our smooth regressors. We start by determining the linear combination of the variables in the panel that maximizes the variance of the common component in the low-frequency band, i.e. the smoothest linear combination. Then we determine another linear combination with the same property under the constraint of orthogonality to the first, and so on. These *generalized principal components* (GPC), denoted by W_{kt}, can be obtained by means of the generalized eigenvectors $v_1, \ldots, v_n$ associated with the generalized eigenvalues $\lambda_1, \ldots, \lambda_n$, ordered from the largest to the smallest, of the pair of matrices $\left(\hat{\Sigma}_\phi,\ \hat{\Sigma}_\chi + \hat{\Sigma}_\xi\right)$, i.e. the vectors satisfying

$$\hat{\Sigma}_\phi v_k = \lambda_k \left(\hat{\Sigma}_\chi + \hat{\Sigma}_\xi\right) v_k, \tag{9}$$

with the normalization constraints $v_k' \left(\hat{\Sigma}_\chi + \hat{\Sigma}_\xi\right) v_k = 1$ (see Anderson, 1984, Theorem A.2.4, p. 590). The eigenvalue λ_k is equal to the ratio of common-low-frequency to total variance explained by the k-th *generalized principal component* W_{kt}.[7] Of course, this ratio is decreasing with k, so that, the greater is k, the less smooth and more idiosyncratic is W_{kt}.

Regarding the projection of c_t on G_F, observe firstly that, since our model has been specified by (8), the first r GPC's span the same space G_F spanned by the first r ordinary principal components (see FHLR, 2005), so that projecting c_t on the first r GPC's would give the same result as projecting on the first r PC's, namely κ_t. However, the variable c_t is by construction very smooth. Therefore its projection on G_F is likely to be well approximated using only the first, and smoother, GPC's. In other words, a fit almost as good as that obtained by the first r ordinary principal components should be obtained by a substantially smoother approximation.

[7]The generalized principal components used in FHLR (2005) are designed for a different purpose. They are obtained using the generalized eigenvectors of the couple $(\hat{\Sigma}_\chi, \hat{\Sigma}_\xi)$.

6 Results

6.1 The number of smooth factors and the definition of NE

Based on the definition of smooth factors and the discussion above, the number of smooth factors is determined by the following procedure:

(I) Firstly we estimate q, the dimension of the white noise u_t, see (7). Applying the criterion proposed in Hallin and Liška (2007), we set $q = 2$. Based on the determination of q, we estimate $\hat{S}_\chi(\theta)$ and $\hat{S}_\xi(\theta)$ as in FHLR (2000, 2005) and compute the covariance matrices $\hat{\Sigma}_\chi$, $\hat{\Sigma}_\phi$ and $\hat{\Sigma}_\xi$ as indicated above.

(II) Secondly, we estimate r, the dimension of G_F, using Bai and Ng's criterion. The result, see Section 5, is $r = 12$. Then, using $\hat{\Sigma}_\chi$, $\hat{\Sigma}_\phi$ and $\hat{\Sigma}_\xi$, we compute the generalized eigenvectors v_k, $k = 1, \ldots, r$ satisfying (9), and the associated GPC's $W_{kt} = v_k' x_t$.

(III) Lastly, let $\kappa_t^{(s)}$ be the projection of c_t on the first s GPC's, while κ_t, as defined in Section 5, is the projection of c_t on the first r principal components (in both cases the principal components are filtered with $(1 + L + L^2)^2$, see Section 5, and the projection is based on (6)). Then let ρ and ρ_s be the R^2's obtained by projecting c_t^* on κ_t and $\kappa_t^{(s)}$ respectively. Starting with $s = 1$, the number of GPC's is increased. We stop when the difference between ρ and ρ_s becomes negligible. Call $\overline{s}$ the number of GPC's so determined.

The fit of the indicator κ_t, i.e. the R^2 of the regression of $c_t^*(T)$ on κ_t, over the period $[13\ \ T-12]$, is 0.77 (see again Section 5 and Table 2). The R^2's of the regression of $c_t^*(T)$ on $\kappa_t^{(s)}$, with s equal to 1, 3, 5, 6, are reported in Table 2. Visual inspection shows that the tiny improvement of the fit between 5 and 6 is not offset by reduced smoothness, thus

Table 2: **Determining the number of generalized principal components**

Number of GPC's	1	3	5	6	
R^2	0.34	0.50	0. 75	0.79	
Number of PC's	1	3	5	6	12
R^2	0.45	0.61	0. 71	0.71	0.77

we set $\bar{s} = 6$.

The projection of c_t on the first 6 GPC's is the New Eurocoin indicator. We use the notation $\hat{c}_t(T)$ for the indicator at time t obtained using the whole sample to estimate the necessary covariance matrices, and $\hat{c}_t(\tau)$ when the subsample $[1\ \tau]$ is used.

New Eurocoin and $c_t^*(T)$ are plotted together in Figure 4 (lower panel, solid line). The advantage of generalized over ordinary principal components, when fit and smoothness are jointly considered, is evident by comparing this to κ_t, Figure 4, upper panel. The next subsection contains a systematic comparison, based on a real-time exercise, of NE to κ_t and $c_t^*(t)$ and the Christiano-Fitzgerald version of the band-pass filter.

6.2 The real-time performance

In this subsection we report a pseudo real-time evaluation of NE. Here "pseudo" refers to the fact that we do not use the true real-time preliminary estimates of the GDP, but the final estimates as reported in GDP "vintage" available in September 2005. The same holds true for all other monthly variables.[8]

[8]A true real time exercise using the different vintages of the data would be preferable. Unfortunately vintages for most of the monthly series in the data set are not available. We prefer the pseudo real time

In Figure 5 and Table 5 we report quantities resulting from the estimation of $\hat{c}_t(\tau)$ and $c_t^*(\tau)$, for some values of τ and t running from November 1998 to June 2005 (the last part of the graph in Figure 5 and the number of consistent signals in Table 5). However, the results using the target $c_t^*(T)$ use only data up to June 2004.

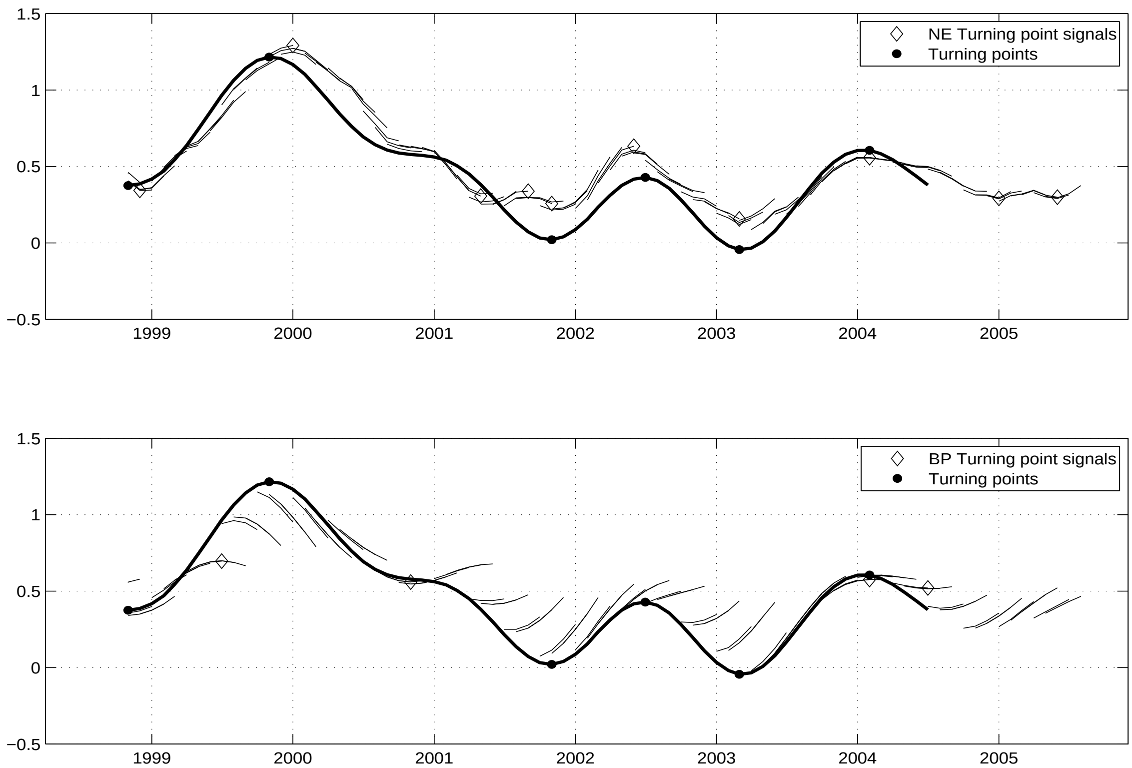

Figure 5: **Pseudo real-time estimates of MLRG, at the end of the sample, obtained with NE (upper panel) and BP (lower panel)**

Intuition for the results presented below is provided in Figure 5. In the upper graph, the long continuous line represents $c_t^*(T)$. The short line ending at t represents the three

exercise rather than resorting to a mixture of latest vintage and real time vintage data, which could produce misleading results.

estimates, $\hat{c}_{t-2}(t)$, $\hat{c}_{t-1}(t)$ and $\hat{c}_t(t)$. Therefore the three points on the short lines over a given t are the first estimate and two revisions of NE at t, namely $\hat{c}_t(t)$, $\hat{c}_t(t+1)$ and $\hat{c}_t(t+2)$. Revisions of NE at t are due to re-estimation of the factors and the projection as new data arrive and are modest. The bullets indicate turning points and the diamonds indicate turning point signals (see below for formal definitions).

The lower graph shows the corresponding estimates for BP. Each short line represents $c^*_{t-2}(t)$, $c^*_{t-1}(t)$ and $c^*_t(t)$. Clearly the band-pass filter estimates (BP), although very smooth, exhibit a large bias towards the sample mean. NE estimates are more accurate and the revision errors are smaller.

Let us now establish the formal criteria used in our evaluation. We are interested in:
(a) the ability of $\hat{c}_t(t) - \hat{c}_{t-1}(t) = \Delta\hat{c}_t(t)$ to signal the correct sign of the change, i.e. the sign of $\Delta c^*_t(T)$, as measured by the percentage of correct signs (see Pesaran and Timmermann, 2006);
(b) the ability of $\hat{c}_t(t)$ to approximate (nowcast) $c^*_t(T)$, for the period $T-81 \leq t \leq T-12$, as measured by the ratio $\sum_{t=T-81}^{T-12}[\hat{c}_t(t) - c^*_t(T)]^2 / \sum_{t=13}^{T-12}[c^*_t(T) - \overline{c^*_t}(T)]^2$, where $\overline{c^*_t}(T) = \sum_{t=13}^{T-12} c^*_t(T)/(T-24)$;
(c) the size of the revision errors after one month, as measured by the ratio $\sum_{t=T-81}^{T-1}[\hat{c}_t(t+1) - \hat{c}_t(t)]^2 / \sum_{t=13}^{T-12}[c^*_t(T) - \overline{c^*_t}(T)]^2$.

Our indicator NE, at time t, is compared, using criteria (a), (b) and (c), to three alternative approximations of $c^*_t(T)$, which use information up to time t, that is BP and:

(CFBP) The optimal approximation to the band-pass filter proposed in Christiano and Fitzgerald (2003). Their filter is applied to the interpolated series $y_t(\tau)$, for τ running from $T-81$ to $T-12$. We use the program recommended by Christiano and Fitzgerald[9] in the stationary version, with a long moving average whose coefficients are obtained by inverting an AR model estimated for the interpolated series $y_t(T-81)$, as defined in Section 3.[10]

(PC) κ_t, i.e. the estimate obtained using ordinary principal components.

All the comparisons reported below are *fair*, in that the same information set is available at any time t for each of the four competing indicators (though different indicators use different subsets).

Table 3: **End of sample performance**

Indicator	% Correct prediction of sign of Δc^*	MS of nowcast error/variance of c^*	MS of revision error/variance of c^*
NE	$0.88^\dagger$	0.13	0.005
BP	0.63	0.32	0.061
CFBP	0.66	0.27	0.133
PC	0.62	0.21	0.116

Notes: Sample November 1998-June 2004. The first column reports the percentage of correct signs with respect to those of Δc^*.

† In this case the null of no predictive power is rejected at 1% significance level (Pesaran and Timmermann, 2006).

Table 3 shows that as far as the criteria (a), (b) and (c) are concerned NE scores better than BP and CFBP, the second outperforming the first as regards the nowcast error and the slope changes.[11] As expected, PC performs fairly well as far as (b) and (c) are concerned[12], but is outperformed by NE by criterion (a). Hence, NE dominates the

[9]The code was downloaded from http://www.clevelandfed.org/research/models/bandpass/bpassm.txt.

[10]The observation in footnote 3 applies to the AR model estimated using the covariances of $y_t(T-81)$.

[11]The advantage of CFBP at the end of the sample vanishes at $T-12$. In other words, the target computed using BP and CFBP are almost identical.

[12]By construction, NE should nowcast as well as PC, hence the better performance of NE in the second

other indicators for the criteria we selected.

As regards the nowcast error of BP and CFBP, we must keep in mind that in the pseudo real-time exercise the delay of the GDP with respect to t can be one, two or three months, see Section 2. The figures 0.32 and 0.27 in the table can be referred to the average delay, which is two months. We should also observe that further publication delay for the GDP may occur in actual NE production.

6.3 The behavior around turning points

The figures in the first column of Table 3, concerning the percentage of correct signs, suggest that NE should perform well in signaling turning points in the target. In the remainder of the present section we explore this issue, but for that purpose we need precise definitions of turning point, turning point signal and false signal.

To begin with, we define a *turning point* as a slope sign change in our target, $c_t^*(T)$. We have an *upturn (downturn)* at time t, if $\Delta c_{t+1}^*(T) = c_{t+1}^*(T) - c_t^*(T)$ is positive (negative), whereas $\Delta c_t^*(T) = c_t^*(T) - c_{t-1}^*(T)$ is negative (positive). According to this definition, in the subsample involved in the pseudo real-time exercise the target exhibits 3 downturns and 3 upturns (see Figure 5).

Next we define a rule to decide when a slope sign change of our indicator $\hat{c}$ can be interpreted as a reliable signal of a turning point in the target c^*. To this end, we focus on the sign of the last two changes of the current estimate of our indicator $\hat{c}_t(t)$, and the

column is due to the particular sample chosen for the real-time exercise.

Table 4: **Classification of signals**

	$\Delta\hat{c}_{t-2}(t-1)$	$\Delta\hat{c}_{t-1}(t-1)$	$\Delta\hat{c}_{t-1}(t)$	$\Delta\hat{c}_{t}(t)$	Consistency	Signal type
1	−	−	−	+	yes	**upturn** at $t-1$
2	+	−	−	+	yes	uncertainty
3	−	−	−	−	yes	deceleration
4	+	−	−	−	yes	slowdown
5	+	+	+	−	yes	**downturn** at $t-1$
6	−	+	+	−	yes	uncertainty
7	+	+	+	+	yes	acceleration
8	−	+	+	+	yes	recovery
9	−	−	+	−	no	trembling deceleration
10	+	−	+	−	no	downturn at $t-2$ shifted
11	−	−	+	+	no	missed upturn
12	+	−	+	+	no	downturn at $t-2$ not confirmed
13	+	+	−	+	no	trembling acceleration
14	−	+	−	+	no	upturn at $t-2$ shifted
15	+	+	−	−	no	missed downturn
16	−	+	−	−	no	upturn at $t-2$ not confirmed

sign of the last two changes of the previous estimate $\hat{c}_t(t-1)$, that is

$$\text{current estimate:} \qquad \cdots \qquad \Delta\hat{c}_{t-1}(t) \qquad \Delta\hat{c}_t(t) \tag{10}$$

$$\text{previous estimate:} \qquad \Delta\hat{c}_{t-2}(t-1) \qquad \Delta\hat{c}_{t-1}(t-1) \qquad \cdots \tag{11}$$

A sign change between $\Delta\hat{c}_{t-1}(t)$ and $\Delta\hat{c}_t(t)$ makes (10) a candidate as a *signal at t locating a turning point at $t-1$*. However, we accept the sign change in (10) as a turning point signal only if (a) the signal is *consistent*, i.e. the signs of $\Delta\hat{c}_{t-1}(t-1)$ and $\Delta\hat{c}_{t-1}(t)$ coincide, and (b) there is no sign change in (11) between $t-2$ and $t-1$, i.e. the signs of $\Delta\hat{c}_{t-2}(t-1)$ and $\Delta\hat{c}_{t-1}(t-1)$ coincide.

The reason for conditions (a) and (b) is that we want to be strict enough to rule out sign changes that may be caused by unstable estimates rather than by true turning points. Condition (a) is obvious. Condition (b) rules out a sign change between $t-1$ and t that follows the opposite change between $t-2$ and $t-1$ in the previous estimate.

Table 4 lists the 8 possible consistent (rows $1-8$) and the 8 possible inconsistent signals (rows $9-16$) which, in principle, could arise. Note that only 2 out of the 8 consistent sign changes in (10) are classified as turning point signals, namely those in the first and the fifth row of Table 4, an upturn and a downturn respectively. Once we have established a rule to identify turning point signals in our indicator we can compare them with turning points that actually occurred in the target.

We say that an upturn (downturn) signal at t locating a turning point at $t-1$ is *false* if c^* has no upturns (downturns) in the interval $[t-3, t+1]$, *correct* otherwise. With this definition, an upturn signal in $\hat{c}_t$ leading or lagging the true upturn (i.e. an upturn in c^*_t) by a quarter or more is false, whereas a two-month error is tolerated.

Table 5: **Real time detection of turning points (TP)**

Target	Consistent signals	Uncertainty signals	TP signals	TP signals excl. last 12 months	Correct TP	Correct over signalled TP	Missed over all TP
NE	81	0	11	8	6	6/8	0
BP	80	0	4	4	1	1/4	5/6
CFBP	68	0	6	6	2	2/6	4/6
PC	81	9	22	20	6	6/20	0

Notes: Sample November 1998-August 2005. The first column reports the number of consistent signals (over a total of 81 signals). The fourth column reports the number of turning point signals when excluding the last 12 signals. The fifth column counts the number of correct turning point signals, i.e. those matching the ones in the target. The last shows the percentage of turning points in the target which are missed by each indicator.

Table 5 shows results for the competing indicators in our real-time exercise. Signals are reported up to the last possible date within our dataset, which is August 2005, although

Interestingly, across methods most signals in real time are consistent, all of them for NE and PC. PC also provides 9 uncertain signals. NE signals 11 turning points (third column), 8 of which before the last 12 months, where c^* is reliable. The latter include all of the 6 turning points in the target. The PC indicator correctly signals all turning points but produces many false signals. By contrast, BP and CFBP produce only a few turning point signals, but most of them are false. Overall, the results on turning points are consistent with the figures in the first columns of Table 3 and, as regards BP, with Figure 5.

6.4 The forecasting properties of the indicator

In Section 3 we argued that we should expect a close match between NE and the GDP growth rate once the latter is smoothed with a moving average such as the one induced by the year-on-year transformation and adjusted for the phase shift.[13] This is confirmed by the results shown in the last two columns of Table 6. While the root mean squared error of NE with respect to quarter-on-quarter GDP growth (first column) is 0.20, the same statistics with respect to year-on-year growth (divided by 4) is 0.18 (second column) and decreases to 0.13 and 0.17 when we adjust for the phase shift by considering future year-on-year growth (third and fourth column).[14] None of the competing indicators have similar forecasting properties.

[13] A similar idea is exploited in Cristadoro et al. (2005) to motivate their result that a core inflation indicator obtained as a smoothed projection of CPI inflation on factors is a good forecaster of the CPI headline inflation.

[14] Obviously, we can compare our monthly indicator with actual GDP growth rates only at the end of each quarter.

Table 6: **How to relate the monthly indicator to actual GDP growth**

	RMSE with respect to different growth rates (%)			
Indicator	Quarter-on-quarter current quarter	Year-on-year current quarter	Year-on-year 1 quarter ahead	Year-on-year 2 quarters ahead
NE	0.20	0.18	0.13	0.17
BP	0.32	0.22	0.22	0.25
CFBP	0.32	0.23	0.23	0.25
PC	0.21	0.17	0.15	0.21

Notes: Sample December 1998-June 2005.

To better gauge the forecasting ability of NE we compare it with univariate ARMA models of quarterly GDP growth, selected by standard in-sample criteria. Such models are often used as benchmarks in forecasting studies (Stock and Watson, 2002b).

Table 7: **Pseudo real-time forecast performance**

	Target growth rates (%)		
Model	Quarter-on-quarter current quarter	Year-on-year current quarter	Year-on-year 1 quarter ahead
NE	0.20	0.18	0.13
AR (AIC)	0.29 ***	0.16	0.17
AR (BIC)	0.29 ***	0.16	0.17
ARMA (AIC)	0.31 **	0.18	0.21 **
ARMA (BIC)	0.30 **	0.17	0.19 *
Random walk	0.31 **	0.18	0.19

Notes: Sample December 1998-June 2005. The first column reports the root mean square forecast error with respect to *current* quarter-on-quarter GDP growth rate, the second with respect to *current* year-on-year GDP growth rate, the third with respect to *next* quarter year-on-year GDP growth rate. NE is the New Eurocoin forecast obtained using the monthly dataset with information updated at most up to last month of the current quarter. The AR and ARMA models are selected at each step according to their in sample performance (in parenthesis the selection criterion used), and are estimated on the quarterly GDP series. A *** , ** or * indicate rejection of the null of equal forecast accuracy at 1, 5 or 10%, respectively, according to Diebold and Mariano (1995) test.

As shown in Table 7, for quarter-on-quarter GDP growth (first column) and for the year-on-year growth rate one quarter ahead (third column) the forecast error of the indicator is far lower than those obtained either with the ARMA or with the random walk.

7 Summary and conclusion

Our coincident indicator NE is a timely estimate of the medium to long-run component of euro area GDP growth. The latter, our target, has been defined as a centered, symmetric moving average of GDP growth, whose weights are designed to remove all fluctuations of period shorter than one year. As observed in Section 3, the target, which has a rigorous spectral definition, leads the "popular" measure of medium to long-run change, namely year-on-year GDP growth, by several months.

We avoid the end-of-sample bias typical of two-sided filters by projecting the target onto suitable linear combinations of a large set of monthly variables. Such linear combinations are designed to discard useless information, namely idiosyncratic and short-run noise, and retain relevant information, i.e. common, cyclical and long-run waves. Both the definition and the estimation of the common, medium to long-run waves are based on recent factor model techniques. Embedding the smoothing into the construction of the regressors is in our opinion an important contribution of the present paper.

The performance of NE as a real-time estimator of the target has been presented in detail in Section 6. The indicator is smooth and easy to interpret. In terms of turning points detection, it scores much better than the competitors that naturally arise as estimators of the medium to long-run component of GDP growth in real time. The reliability of the signal is reinforced by the fact that the revision error of our indicator is fairly small as compared with the competitors. We have also shown that NE is a very good forecaster of year-on-year GDP growth 1 and 2 quarters ahead; it also scores well in forecasting quarter-on-quarter GDP growth, with an RMSE of 0.20, which ranks well even in comparison with best practice results.

Appendix

A.1 Computing $c_t^*(T)$. Linear interpolation

In Section 3 we claim that the particular technique chosen to interpolate the missing observations in the GDP growth is not likely to make a serious difference *as far as* c_t *is concerned.* The following experiment gives an idea of the effect of linearly interpolating the missing observations. We take a monthly series, compute its monthly quarter-on-quarter growth rate z_t, compute the linearly interpolated series Z_t, as though z_t were not observable for the first two months of each quarter, and compare $a_t = \beta(L)z_t$ with $b_t = \beta(L)Z_t$. For the industrial production index of the euro zone, $\text{var}(b_t - a_t)/\text{var}(a_t) = 0.004$. Similar results were obtained for other series and by applying Chow and Lin's method instead of linear interpolation. This is hardly surprising. Our monthly quarter-on-quarter growth-rate series have by construction a strong, positive autocorrelation at the first lags, due to overlapping (see Section 2), so that linear interpolation, as well as Chow and Lin's, should not be so far from actual data. The remaining difference is made up of short-run oscillations that, as already observed in Section 3, are smoothed off by the filter $\beta(L)$.

A.2 The approximation error $c_t - c_t^*(T)$. Simulation results

We start with the observed *quarterly* series of the GDP growth rate, whose sample length is 73. Then:

(I) Having selected (AIC), among several low-order ARMA models, an AR(1), we estimate the autoregressive polynomial $b(L) = 1 - 0.51\,L$.

(II) For each replication we use $b(L)$ to generate an AR(1) of length $m = 2n + 73 + 2n$, which is thought of as a quarterly series. The latter is linearly interpolated to obtain a "monthly" series of length $M = 3m - 2 = 217 + 12n$. Denote by $c^*_{j,t}(M)$ the corresponding band-passed series. It is convenient to assume that t runs from $-6n + 1$ to $217 + 6n$.

(III) A preliminary simulation exercise determines M, or, which is equivalent, n. Precisely, n is chosen in such a way that

$$\max_{t=1,\ldots,217} \max_{j=1,\ldots,2000} \frac{[c^*_{j,t}(217 + 12n) - c^*_{j,t}(217 + 6n)]^2}{\text{var}(c_{j,t}(217 + 12n))} \leq 0.01.$$

We find that $n = 200$ is sufficiently large. As a consequence, for $\hat{M} = 217 + 1200$ and t belonging to the central subsample of length $T = 217$, we set $c_{j,t} = c^*_{j,t}(\hat{M})$.

(IV) The mean and the median over 2000 replications of the ratio $v_{j,t,T}$, as defined in Section 3, are reported in Table 8.

Table 8: **Mean and median of the ratio** $v_{j,t,T}$

	$t = T$ Typical	$t = T$	$t = T - 12$	$t = T - 108$
Mean	0.45	0.14	0.008	0.0013
Median	0.30	0.06	0.003	0.0006

The first two columns require an explanation. As we have observed in Section 2, the average delay between the last publication date of the GDP and the current time is 2. Therefore at $t = 219$, the last publication date being at 217, we have the typical situation. This is the first column. The second column represents the artificial situation in which the GDP of quarter [215, 216, 217] is published in month 217.

At $T - 12$ the approximation is quite good on average, with only 4% of the values of $v_{j,T-12,T}$ exceeding 0.035. This, jointly with the results in Appendix A.3 supports our identification of c_t^* and c_t for $13 \leq t \leq T-12$.

Regarding the end-of-sample deterioration, the figures 0.45 in Table 8 and 0.30 in Table 3 (corresponding to BP in the second column), can be compared, the first being the average over different replications, the second a time average within one realization. Among possible explanations for the difference, observe that in the subsample used in the real-time exercise the GDP is closer to its mean, relative to the whole sample.

A.3 The approximation error $c_t - c_t^*(T)$. Exact results

The size of the approximation error $c_t - c_t^*(T)$, as measured by $\mathrm{E}[(c_t - c_t^*(T))^2]/\mathrm{var}(c_t)$, can be computed exactly under the assumption that y_t is a known monthly stationary process. In this case no linear interpolation is necessary and our construction of $c_t^*(T)$, see Section 3, only requires application of the band-pass filter to the series y_t, observed in $[1\ T]$ and augmented with its mean $\hat{\mu}$ outside $[1\ T]$. Noting that $\hat{\mu} = \frac{1}{T}\sum_{k=t-T}^{t-1} y_{t-k}$ and that $\beta(1) = \sum_{-\infty}^{\infty} \beta_k = 1$, we have:

$$c_t^*(T) = \sum_{k=-\infty}^{t-T-1} \beta_k \hat{\mu} + \sum_{k=t-T}^{t-1} \beta_k y_{t-k} + \sum_{k=t}^{\infty} \beta_k \hat{\mu} = \sum_{k=t-T}^{t-1} \beta_k^{(t,T)} y_{t-k},$$

where

$$\beta_k^{(t,T)} = \beta_k + \frac{1}{T}\left(1 - \sum_{k=t-T}^{t-1} \beta_k\right).$$

The filter $\beta^{(t,T)}(L)$, obtained by truncating and correcting $\beta(L)$, is finite and only requires the sample y_t, $t = 1, \ldots, T$.

We have:

$$c_t - c_t^*(T) = (\beta(L) - \beta^{(t,T)}(L))y_t,$$

so that, if g_y is the spectral density of y_t,

$$\frac{\mathrm{var}(c_t - c_t^*(T))}{\mathrm{var}(c_t)} = \frac{\int_{-\pi}^{\pi} \left|\beta(e^{-i\theta}) - \beta^{(t,T)}(e^{-i\theta})\right|^2 g_y(\theta)d\theta}{\int_{-\pi}^{\pi} |\beta(e^{-i\theta})|^2 g_y(\theta)d\theta}. \tag{12}$$

Note that, as $\mathrm{E}(c_t) = \mathrm{E}(c_t^*(T)) = \mathrm{E}(y_t)$, the numerator in (12) is equal to $\mathrm{E}[(c_t - c_t^*(T))^2]$. Thus (12) is comparable to the results in Appendix A.2 and in Section 6. In Table 9 we report the value of the ratio (12) for several processes $y_t = a(L)u_t$, where u_t is a white noise. We set $T = 217$, the size of our dataset, and compute (12) for $t = T - 12$ and $t = T - 108$, which is the middle of the sample.[15]

Table 9: **Ratio** $\mathrm{var}(c_t - c_t^*(T))/\mathrm{var}(c_t)$

$y_t = a(L)u_t$	$t = T - 12$	$t = T - 108$
$a(L) = 1$	0.027	0.006
$a(L) = 1 + .9L$	0.025	0.005
$a(L) = 1 - .6L$	0.040	0.008
$a(L) = (1 - .8\,L)^{-1}$	0.010	0.002
$a(L) = [(1 - .8L)(1 + .4L)]^{-1}$	0.010	0.002

We see that the approximation is good within the sample up to $T - 12$ for the positively autocorrelated processes, reasonable for the negatively autocorrelated moving average.[16] The figures corresponding to the autoregressive process $(1 - 0.8\,L)^{-1}u_t$ and those obtained in the simulation are very close, see the first row in Table 8 (third and fourth columns)

[15]Because of the linear interpolation used in Appendix A.2, the values of (12) for $t = T + 2$ and $t = T$ cannot be compared to the corresponding figures in Table 8 and are therefore not reported.

[16]The integrals in (12) have been computed using the Matlab routine "quad". The results do not differ significantly from those obtained in the time domain using very long approximations of the filters involved.

and the fourth row in Table 9.[17]

A.4. Transforming the factors with $(1 + L + L^2)^2$

Suppose for the moment that the variables in the dataset are all obtained as first differences, $x_{it} = X_{it} - X_{it-1}$, so that the factors $F_{1t},\ F_{2t},\ \ldots,\ F_{rt}$ are linear combinations of first differences. The natural specification of the assumption that the GDP is driven by the factors F_{kt}, see Section 5, is that the month-on-month change of the GDP (unobservable) can be represented as:

$$x_t = (1 - L)X_t = \chi_t + \xi_t = c_1 F_{1t} + c_2 F_{2t} + \cdots + c_r F_{rt} + \xi_t,$$

with ξ_t orthogonal to the factors and ξ_{it} at any lead and lag for all i. Then note that the quarter-on-quarter GDP change is

$$\begin{aligned} y_t &= (X_t + X_{t-1} + X_{t-2}) - (X_{t-3} + X_{t-4} + X_{t-5}) = (1 + L + L^2)^2 x_t \\ &= c_1(1 + L + L^2)^2 F_{1t} + \cdots + c_r(1 + L + L^2)^2 F_{rt} + (1 + L + L^2)^2 \xi_t. \end{aligned}$$

The assumption that ξ_t is orthogonal to the factors at any lead and lag implies that

$$c_1(1 + L + L^2)^2 F_{1t} + \cdots + c_r(1 + L + L^2)^2 F_{rt}$$

is the orthogonal projection of y_t on the factors, and also the projection of y_t on the factors transformed by $(1 + L + L^2)^2$, the same holding for c_t, which is a linear combination of leads and lags of y_t.

[17] However, as already observed in footnote 3, the series used in the simulation, being obtained by linear interpolation, are not covariance stationary, so that the comparison above has no more than a heuristic interpretation.

However, the argument above can only hold approximately in our empirical situation. As a matter of fact, the majority of the variables in our dataset, as well as y_t, are rates of change, not changes, and a few of them do not even need a transformation to achieve stationarity (see Appendix B). On the other hand, running the two regressions: (i) c_t on the factors transformed by $(1 + L + L^2)^2$, (ii) c_t on contemporaneous and past values of the factors, we find that they do not differ much in terms of fitting but that the first is smoother. Therefore we choose the first option for the regressors.

B. Dataset and treatment

The dataset includes 145 series from Thomson Financial Datastream, referring to the euro area as well as its major economies. The final dataset used in this paper is the result of a search process in a much larger dataset of euro area and national variables. Criteria for choice are: (i) a sufficient time series span (at least starting in 1987), (ii) a high correlation with respect to GDP growth, (iii) early release date. Moreover, the dataset includes groups of variables that are, according to current practice in conjunctural analysis, leading, lagging and coincident with respect to the GDP. In particular, the presence of leading variables, which contain information about future values of the GDP, is crucial to obtain a good estimate of c_t at the end of the sample (see the Introduction and Section 5).

For the euro area GDP we use data from Fagan et al. (2001) until the first quarter 1991, from then on we use the official Eurostat series (rescaling data prior to 1992 to avoid a sudden change in level). The database is organized into homogeneous blocks, i.e. industrial Production Indexes (41 series), Prices (24), Money Aggregates (8), Interest

Rates (11), Financial Variables (6), Demand Indicators (14), Surveys (25), Trade Variables (9) and Labour Market Series (7).

All series are transformed to remove outliers, seasonal factors and non-stationarity. Regarding outliers, we eliminate from each series those observations that are more than 5 standard deviation away from the mean and replace them with the sample average of the remaining observations. Seasonal adjustment is obtained by regressing the variables on a set of seasonal dummies. We do not resort to other more sophisticated procedures (e.g. Tramo-Seats or X12) to avoid the use of two-sided filters, which would potentially imply large revisions in the seasonally adjusted series.

The results of unit-root tests are homogeneous for variables belonging to a given economic class (e.g. industrial production, prices and so on), with only a few exceptions. For 9 out of our 145 series stationarity is achieved by $1-L$ (Interest Rates), 26 do not need a transformation (Surveys + one Interest Rate), the others are transformed by $(1-L)\log$.

Lastly, the series are normalized subtracting the mean and dividing for the standard deviation as usually done in the large factor model literature. A detailed list of the variables and related transformations is available upon request.

Appendix C: New and Old Eurocoin

Leaving aside minor improvements in the dataset and the realignment technique, the main changes are: (1) The target of NE is c_t, the medium to long-run component of the GDP. As we argue in the paper, c_t can be approximated with good accuracy for $13 \leq t \leq T-12$. The target in Old Eurocoin was the medium to long-run component of the common component of the GDP. The latter can be approximated only for n, the number of series in the dataset, tending to infinity. As a consequence, Old Eurocoin had no measure of performance. (2) Moreover, Old Eurocoin uses the regressors of FHLR (2005), which are generalized principal components maximizing the common variance. Though smoother than the ordinary PC's, still they contain a sizable short-run component. In the present paper we use the smooth regressors defined in Section 5. As a result NE is smoother than Old Eurocoin and the number of false turning point signals is significantly reduced.

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