The Costs and Benefits of Informalization in a Two-Sector New Keynesian Model *

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Abstract

This paper explores the costs and benefits of informalization of the economy in a New Keynesian two-sector closed economy. The informal sector is more labour intensive, is untaxed and has a classical labour market. We consider the case where it is also a 'hidden' unobserved sector. The more capital intensive formal sector bears all the taxation costs and wages are determined by a real wage norm. We identify three welfare costs of informalization: (1) Long-term costs of a steady state where taxes are restricted to the formal sector (2) Short-term fluctuation costs of changes to taxes to finance fluctuations in government spending again restricted to the formal sector and (3) The costs associated with lack of observability of the informal sector. These mean that inflation targeting using the interest rate can only target inflation in the formal sector. The benefit of an informal sector that is characterized by low labour market frictions derives from a possible contribution to lower business cycle costs. We investigate the circumstances under which taxation of the informal sector that reduces its size sees a net welfare improvement with a reduction in costs (1) to (3) outweighing a possible stabilization benefit.

JEL Classification: J65, E24, E26, E32

Keywords: Informal economy, labour market, tax policy, interest rate rules, inflation targets

Contents

1 Introduction

A large informal sector is an important characteristics in many countries and the study of informality can shed new lights on the impact of labour market and monetary policies on the economic performance of this economies. In particular, the informal economy is important in developing countries and also in many transitional and some OECD countries and if we want to analyse the transmission mechanism of monetary policy we need to model the informal labour market in an explicit way.

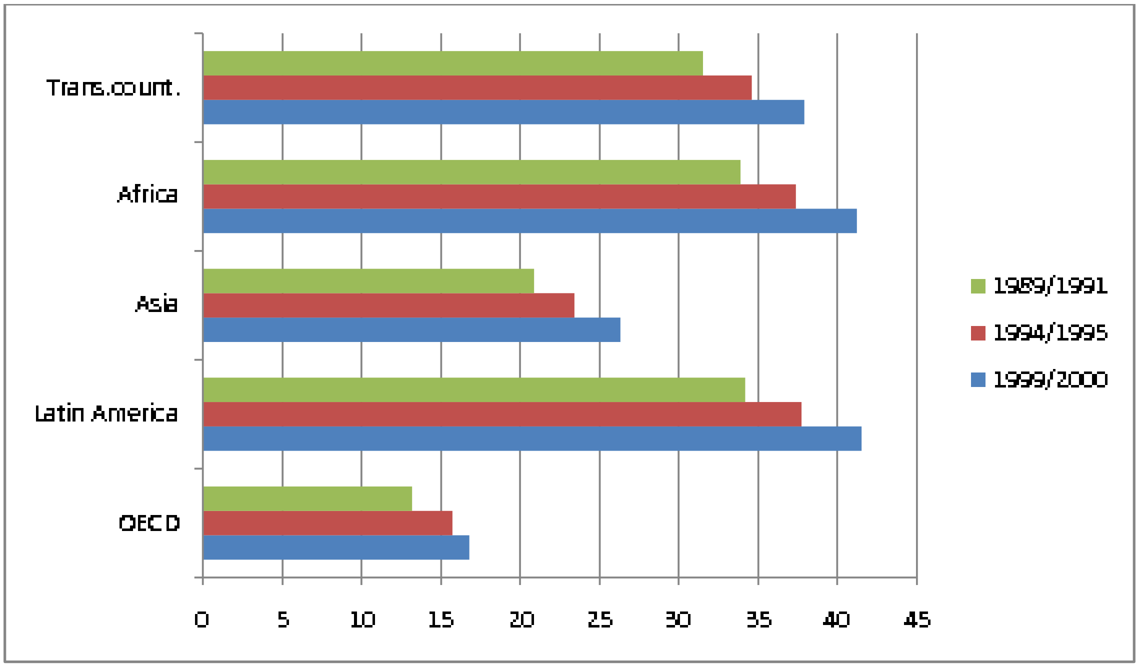

Figure 1: **Size of Informal Economy around the World Schneider (2005)**

The phenomenon, that we refer in our paper as 'informality' has been discussed using different terminology: unregistered, hidden, shadow, unofficial underground and, in a more restrictive sense black, economy. Different terms may be more or less correct depending on the context. For example, the term 'hidden economy' has often been used with respect to advanced economies, while the term 'informal economy' has been usually described with reference to developing economies. Chen (2007) describes the move from the 'old' concept of the informal economy to a more comprehensive view of informality. The 'new' view of informality which focuses on the worker and informal employment, that is employment without any sort of protection, includes self-employment in unregistered firms and wage employment in unprotected jobs. According to the definitions used, the estimates of the size of the informal economy can be very different.[1]

In general, different modelling strategies apply to describe different phenomena. It is possible to distinguish informality in the good market, informality in the credit market and informality in the labour market.[2] Here we focus on the latter, i.e., the labour market aspects of informality, and study the impact of monetary policy in an economy where the size of the informal sector depends on the taxation regime. In our model public goods are produced formally and the two sectors have different technologies, the informal sector being more labour intensive. A further distinction is that we introduce market friction in the labour market in the formal sector, whilst the informal sector is frictionless in this respect. Thus we capture some of the main characteristics of the informal sector: labour-intensiveness, lack of public goods production and invisibility. Price stickiness is added to both sector to give us the New Keynesian aspect and a model that can be used to investigate the flows between formal and informal sector and the link between inflation and unemployment, i.e. the Phillips curves in countries with a large informal economy. We study optimal monetary policy and consider the extent to which the difficulty in observing the informal sector affects the its efficacy.

The remainder of the paper is organized as follows. Section 2 shows how our general equilibrium economy relates to similar theoretical frameworks within the DSGE and the informal economy literatures. Section 3 sets out details of our model. Section 4 studies optimal monetary policy and finally, section 5 concludes.

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[1] Informality is also defined in different ways in various countries. For example, in India, the informal sector is generally identified with the unorganized sector (no legal provision and no regular accounts). In particular, according to the NCEUS report on Definitional and Statistical Issues relating to the Informal Economy, the informal (or unorganized) economy is given by the informal (or unorganized) sector and its workers plus the informal workers in the formal sector, where the unorganized (informal) sector is defined as all incorporated private enterprises owned by individuals/households with less than 10 workers. Also the report defines unorganized (informal) workers, as workers in the unorganized sector, households, excluding regular workers with social security benefits, plus workers in the formal sector without any social security benefit NCEUS (2008).

[2] See Batini *et al.* (2009) for details.

2 Background Literature

Satchi and Temple (2009) and Marjit and Kar (2008) recognize the importance of a *general equilibrium analysis*, but they often make assumptions which usually do not allow for a comprehensive view of causes and effects of informality. For example, standard assumptions in the search-matching literature (i.e. linear utility function among the other), exclude the consumption-hours decision. Conesa *et al.* (2002) represents an attempt in this direction. The author describes a simple RBC model with an informal sector. They introduce a second sector into a standard Real Business Cycle (RBC) model which is described as an "underground" economy that has a different technology, produces goods and services that could otherwise be produced in the formal sector, but is not registered in NI accounts. The main characteristics of the model includes: a wage premium which can be seen as the opportunity cost of not working in the official sector and labour indivisibilities in the formal/registered sector. Households choose a probability of working in the informal sector which can be interpreted as the purchase of lotteries in a perfectly insured market. When a worker chooses the informal sector he/she enjoy more leisure at the price of a smaller wage, while in the formal sector individuals work more, but receive a wage premium. In particular, the authors assume labour to be indivisible in the formal/registered sector with hours worked fixed exogenously. The main prediction of the model is that wage premium differentials can explain the different size of macroeconomic fluctuations in function of technological shocks. The intuition is the following: countries with a smaller wage premium have a lower opportunity cost to participate in the formal sector and so they have smaller participation rates. In those countries, the effects of technological shocks are amplified.

A series of papers incorporate the search and matching approach into DSGE models to explain the cyclical behaviour of employment, job creation, job destruction and inflation rate in response to a monetary policy shock .[3] In general there is a rapidly growing literature on search and match labour market in New Keynesian DSGE models in addition to Ravenna and Walsh (2007). Christiano *et al.* (2007), Sala *et al.* (2008) and Thomas (2008) introduce labour market frictions in New Keynesian models allowing the study of,

[3]See Yasgiv (2007) for a survey on the developments of search-matching models and (Ravenna and Walsh (2007)) for a recent application of search frictions in New Keynesiam models.

both, the intensive and the extensive margin of labour usage during the business cycle. Blanchard and Gali (2007) adopt a simpler hiring cost approach in a New Keynesian framework.

Castillo and Montoro (2008) develop Blanchard and Gali (2007) by modelling a labour market economy with formal and informal labour contracts within a New Keynesian model with labour market frictions. This is the first paper that analyses together the creation of informal jobs and the interaction between the informal sector and monetary policy. Informality is a result of hiring costs, which are a function of the labour market tightness. In equilibrium , firms in the wholesale sectors balance the higher productivity of a formal production process with the lower hiring costs of the informal process. Marginal costs will then become a function also of the proportion of informal jobs in the economy. The interesting results of this theoretical framework is that during period of high aggregate demand the informal sector expands due to lower hiring costs associated with this technology. This creates a link between informality, the dynamics of inflation and the transmission mechanism of monetary policy. In particular, the authors show that *"informal workers act as a buffer stock of labour that allows firms to expand output without putting pressure on wages"*.. Castillo and Montoro (2008) allow for a voluntary decision where the marginal worker is indifferent between formal and informal sector. Labour market regulations may reduce labour demand without introducing segmentation per se. This picture is realistic in many advanced economies and there is also evidence that shows the existence of a voluntary, small firms sector in some developing countries Perry *et al.* (2007).

Two related papers are Thomas (2008) and Zenou (2008). We adopt a NK framework with extensive and intensive margins as in Thomas (2008) and allow for a frictionless informal labour market as in Zenou (2008). We also introduce labour market frictions in the informal sector, but we do not explicitly model the matching process as in these papers. Rather we follow another modelling option favoured in the literature (see Blanchard and Gali (2007) for a discussion of this option and why it is empirically plausible) by introducing a wage norm in the formal sector. As clarified in Zenou's paper *".. in the informal sector, either people are self-employed or work with relatives or friends and thus do not apply formally for jobs posted in newspapers or employment agency"*. As in Zenou (2008) we do not model this idea explicitly, but our competitive informal labour market implies free-entry and an instantaneous hiring process. Zenou's framework has no NK features and focuses on the evaluation of various labour market policies on the unemployment rate of an economy with an informal sector.

Our paper contribution to this literature is as follows. We look at the efficacy of monetary policy and for this reason we require a more general framework where households' consumption and leisure decisions are explicitly modelled in a DSGE model with price rigidity. We introduce New Keynesian rigidities in the usual way, as in Castillo and Montoro (2008), but then proceed analyse the interaction of informal and formal sectors and the implications for monetary policy. Our analysis of simple optimized Taylor-type interest rate rules, the incorporation of zero-lower bound constraint and the comparison between simple and optimal rules where one sector is unobserved are particularly novel features for the informal economy literature.

3 The Model

Consider a two-sector "Formal" (F) and "Informal" (I) economy, producing different goods with different technologies which sell at different retail prices, $P_{F,t}$ and $P_{I,t}$, say. Labour and capital are the variable factor inputs and the formal sector is less capital intensive. Government spending is financed by an employment tax as in Zenou (2008). In the general set-up this can be shared by the formal and informal sectors giving us a framework in which the role of tax incidence can be studied as one of the drivers of informalization.

3.1 Households

A proportion $n_{F,t}$ of household members work in the formal sector. Hours $h_{F,t}$ and hours $h_{I,t}$ are supplied in the F and I sectors respectively. Members who work in sector $i = I, F$ derive utility $U(C_t, L_{i,t})$ where C_t is household consumption and leisure $L_{i,t} = 1 - h_{i,t}$ and we assume that[4]

$$U_C > 0,\ U_L > 0,\ U_{CC} \leq 0,\ U_{LL} \leq 0 \tag{1}$$

. The representative household single-period utility is

$$\Lambda_t = \Lambda(C_t, n_{F,t}, h_{F,t}, h_{I,t}) = n_{F,t} U(C_t, 1 - h_{F,t}) + (1 - n_{F,t}) U(C_t, 1 - h_{I,t}) \tag{2}$$

[4]Our notation is $U_C \equiv \frac{\partial U}{\partial C}$, $U_{CC} \equiv \frac{\partial^2 U}{\partial C^2}$ etc; $U_{L_{F,t}} \equiv \frac{\partial U(C_t, L_{F,t})}{\partial L_{F,t}}$, etc.

We construct Dixit-Stiglitz consumption and price aggregates

$$C_t = \left[\mathrm{w}^{\frac{1}{\mu}} C_{F,t}^{\frac{\mu-1}{\mu}} + (1-\mathrm{w})^{\frac{1}{\mu}} C_{I,t}^{\frac{\mu-1}{\mu}} \right]^{\frac{\mu}{\mu-1}} \quad (3)$$

$$P_t = \left[\mathrm{w}(P_{F,t})^{1-\mu} + (1-\mathrm{w})(P_{I,t})^{1-\mu} \right]^{\frac{1}{1-\mu}} \quad (4)$$

Then standard inter-temporal and intra-temporal decisions lead to

$$\frac{\Lambda_{C,t}}{P_t} = \beta E_t \left[(1+R_{n,t}) \frac{\Lambda_{C,t+1}}{P_{t+1}} \right] \quad (5)$$

$$C_{F,t} = \mathrm{w} \left(\frac{P_{F,t}}{P_t} \right)^{-\mu} C_t \quad (6)$$

$$C_{I,t} = (1-\mathrm{w}) \left(\frac{P_{I,t}}{P_t} \right)^{-\mu} C_t \quad (7)$$

where $R_{n,t}$ is the nominal interest rate over the interval $[t, t+1]$ for riskless bonds set by the central bank at the beginning of the period. Note that substituting (6) and (7) into (3) gives (4) so that (4) or (3) are superfluous for the set-up. Total labour supply is found by equating the marginal rate of substitution between labour and leisure with the real wages for the two sectors:

$$\frac{U_{L_I,t}}{\Lambda_{C,t}} = \frac{W_{I,t}}{P_t} \quad (8)$$

$$\frac{U_{L_F,t}}{\Lambda_{C,t}} = \frac{W_{F,t}}{P_t} \quad (9)$$

We assume that the real wage in the formal sector is a combination of a real wage norm, RW_t and the market-clearing real wage in the informal sector:

$$\frac{W_{F,t}}{P_t} = RW_t^{1-\omega} \left(\frac{W_{I,t}}{P_t} \right)^{1-\omega} \quad (10)$$

We assume that $RW_t > \frac{W_{I,t}}{P_t}$. It follows from $U_{LL} < 0$ that the household will choose less leisure and more work effort in the formal sector; i.e., $h_{F,t} > h_{I,F}$.

3.2 Wholesale Firms

Wholesale output in the two sectors is given by a Cobb-Douglas production function

$$Y_{i,t}^W = F(A_{i,t}, N_{i,t}, K_{i,t}), \ i = I, F \quad (11)$$

where $A_{i,t}$ are a technology, total labour supply $N_{i,t} = n_{i,t} h_{i,t}$, $i = I, F$. Capital inputs are $K_{i,t}$, $i = I, F$ and we assume capital is accumulated from formal output only.

The first-order conditions are

$$P^W_{F,t}F_{N_F,t} = W_{F,t} + P_t\tau_{F,t} \tag{12}$$

$$P^W_{I,t}F_{N_I,t} = W_{I,t} + P_t\tau_{I,t} \tag{13}$$

$$P^W_{I,t}F_{K_I,t} = P^W_{F,t}F_{K_F,t} = R_t + \delta \tag{14}$$

where $P^W_{F,t}$ and $P^W_{I,t}$ are wholesale prices, $\tau_{F,t}$, $\tau_{F,t}$ are the employment real tax rates in the formal sector and informal sectors respectively and $R_t + \delta$ is the cost of capital, the ex post real interest rate over the interval $[t-1, t]$ plus the depreciation rate. R_t is defined by

$$1 + R_t = \left[(1 + R_{n,t-1})\frac{P_{t-1}}{P_t}\right] \tag{15}$$

where $R_{n,t}$ is the nominal interest charged on loans made in period t.

3.3 Retail Firms

We now introduce a retail sector of monopolistic firms within each sector buying wholesale goods and differentiating the product at a proportional resource cost $c_iY^W_{i,t}$ in sectors $i = F, I$. In a free-entry equilibrium profits are driven to zero. Retail output for firm f in sector is then $Y_{i,t}(f) = (1 - c_i)Y^W_{i,t}(f)$ where $Y^W_{i,t}$ is produced according to the production technology (11) at prices $P^W_{i,t}$. nt ta Let the number of differentiated varieties produced in the informal and formal sectors be ν_F and ν_I respectively. Each is produced by a single retail firm and the numbers of these firms is fixed.[5] Let $C_{F,t}(f)$ and $C_{I,t}(f)$ denote the home consumption of the representative household of variety f produced in sectors F and I. Aggregate consumption of each category now become indices

$$C_{F,t} = \left[\left(\frac{1}{\nu_F}\right)^{\frac{1}{\zeta_F}} \sum_{f=1}^{\nu_F} C_{F,t}(f)^{(\zeta_F-1)/\zeta_F}\right]^{\zeta_F/(\zeta_F-1)} \tag{16}$$

$$C_{I,t} = \left[\left(\frac{1}{\nu_I}\right)^{\frac{1}{\zeta_I}} \left(\sum_{f=1}^{\nu_I} C_{I,t}(f)^{(\zeta_I-1)/\zeta_I}\right)\right]^{\zeta_I/(\zeta_I-1)} \tag{17}$$

[5]This model structure closely follows a model of two interacting economies in the New Open Economy Literature.

where $\zeta_F, \zeta_I > 1$ are the elasticities of substitution between varieties in the two sectors. Aggregate output is similarly defined:

$$Y_{F,t} = \left[\left(\frac{1}{\nu_F} \right)^{\frac{1}{\zeta_F}} \sum_{f=1}^{\nu_F} Y_{F,t}(f)^{(\zeta_F - 1)/\zeta_F} \right]^{\zeta_F/(\zeta_F - 1)} \quad (18)$$

$$Y_{I,t} = \left[\left(\frac{1}{\nu_I} \right)^{\frac{1}{\zeta_I}} \left(\sum_{f=1}^{\nu_I} Y_{I,t}(f)^{(\zeta_I - 1)/\zeta_I} \right) \right]^{\zeta_I/(\zeta_I - 1)} \quad (19)$$

Then the optimal intra-sectoral decisions are given by standard results:

$$C_{F,t}(f) = \left(\frac{P_{F,t}(f)}{P_{F,t}} \right)^{-\zeta_F} C_{F,t} \quad (20)$$

$$C_{I,t}(f) = \left(\frac{P_{I,t}(f)}{P_{I,t}} \right)^{-\zeta_I} C_{I,t} \quad (21)$$

and inter-sector decisions are as before.

We introduce endogenous investment, I_t, and exogenous government spending G_t both assumed to consist entirely of formal output. Then maximizing the investment and government expenditure indices as for the consumer in (20) we have

$$I_t(f) = \left(\frac{P_{F,t}(f)}{P_{F,t}} \right)^{-\zeta_F} I_t \quad (22)$$

$$G_t(f) = \left(\frac{P_{F,t}(f)}{P_{F,t}} \right)^{-\zeta_F} G_t \quad (23)$$

Using (20)–(23) it follows that total demands for each differentiated product are given by

$$Y_{F,t}(f) = C_{F,t}(f) + I_t(f) + G_t(f) = \left(\frac{P_{F,t}(f)}{P_{F,t}} \right)^{-\zeta_F} (C_{F,t} + I_t + G_t) = \left(\frac{P_{F,t}(f)}{P_{F,t}} \right)^{-\zeta_F} Y_{F,t} \quad (24)$$

$$Y_{I,t}(f) = C_{I,t}(f) = \left(\frac{P_{I,t}(f)}{P_{I,t}} \right)^{-\zeta_I} C_{I,t} = \left(\frac{P_{I,t}(f)}{P_{I,t}} \right)^{-\zeta_I} Y_{I,t} \quad (25)$$

Retail firms follow Calvo pricing. In sector $i = F, I$, assume that there is a probability of $1 - \xi_i$ at each period that the price of each good f is set optimally to $\hat{P}_{i,t}(f)$. If the price is not re-optimized, then it is held constant.[6] For each producer f the objective is at time t to choose $\hat{P}_{i,t}(f)$ to maximize discounted profits

$$E_t \sum_{k=0}^{\infty} \xi_i^k D_{t,t+k} Y_{i,t+k}(f) \left[\hat{P}_{i,t}(f) - P_{i,t+k} \mathrm{MC}_{i,t+k} \right]$$

[6]Thus we can interpret $\frac{1}{1-\xi_i}$ as the average duration for which prices are left unchanged in sector $i = F, I$.

where $D_{t,t+k}$ is the discount factor over the interval $[t, t+k]$, subject to a downward sloping demand from consumers of elasticity ζ_i given by (24) and (25), and $MC_{i,t} = \frac{P^W_{i,t}}{P_{i,t}}$ are real marginal costs. The solution to this is

$$E_t \sum_{k=0}^{\infty} \xi_i^k D_{t,t+k} Y_{i,t+k}(f) \left[\hat{P}_{i,t}(f) - \frac{\zeta_i}{(\zeta_i - 1)} P_{i,t+k} \text{MC}_{i,t+k} \right] = 0 \tag{26}$$

and by the law of large numbers the evolution of the price index is given by

$$P_{i,t+1}^{1-\zeta_i} = \xi_i \left(P_{i,t}\right)^{1-\zeta_i} + (1-\xi_i)(\hat{P}_{i,t+1}(f))^{1-\zeta_i} \tag{27}$$

These summations can be expressed as difference equations as follows. First define for $i = I, F$, $\Pi_{i,t} \equiv \frac{P_{i,t}}{P_{i,t-1}} = \pi_{i,t} + 1$. Then from the Euler equation we have that $D_{t+k,t} = \beta^k \frac{U_{C,t+k}}{U_{C,t}}$. Using this result we can derive the aggregate price dynamics for $i = I, F$ as

$$H_{i,t} - \xi_i \beta E_t[\Pi_{i,t+1}^{\zeta_i - 1} H_{i,t+1}] = Y_{i,t} U_{C,t} \tag{28}$$

$$J_{i,t} - \xi_i \beta E_t[\Pi_{i,t+1}^{\zeta_i} J_{i,t+1}] = \left(\frac{1}{1 - \frac{1}{\zeta_i}} \right) Y_{i,t} U_{C,t} MC_{i,t} \tag{29}$$

$$\frac{\hat{P}_{i,t}}{P_{i,t}} H_{i,t} = J_{i,t} \tag{30}$$

$$1 = \xi_i \Pi_t^{\zeta_i - 1} + (1-\xi_i) \left(\frac{\hat{P}_{i,t}}{P_{i,t}} \right)^{1-\zeta_i} \tag{31}$$

3.4 Equilibrium

Assuming Cobb-Douglas technology in the wholesale sectors (see all functional forms below) for each differentiated product in the F and I sectors we equate supply and demand in the retail sectors to give

$$Y_{F,t}(f) = (1-c_i) F(A_{F,t}, N_{F,t}(f), K_{F,t}(f)) = \left(\frac{P_{F,t}(f)}{P_{F,t}} \right)^{-\zeta_F} Y_{F,t} \tag{32}$$

$$Y_{I,t}(f) = (1-c_i) F(A_{I,t}, N_{I,t}(f), K_{I,t}(f)) = \left(\frac{P_{I,t}(f)}{P_{I,t}} \right)^{-\zeta_I} Y_{I,t} \tag{33}$$

using (24) and (25). Then solving for $N_{i,t}$, $i = F, I$ and defining aggregate employment-hours in each sector by $N_{i,t} = \sum_{j=1}^{\nu_i} N_{i,t}(j)$, $i = F, I$ we arrive the aggregate production functions

$$Y_{i,t} = \frac{(1-c_i) A_{i,t} (N_{i,t})^{\alpha_i}}{\Delta_{i,t}} \; ; \; i = F, I \tag{34}$$

where

$$\Delta_{i,t} = \sum_{j=1}^{\nu_i} \left(\frac{P_{i,t}(f)}{P_{i,t}} \right)^{-\frac{\zeta_i}{\alpha_i}} \tag{35}$$

is a measure of the price dispersion across firms in sector $i = F, I$. Then the aggregate equilibrium conditions in each retail sector are

$$\begin{aligned} Y_{F,t} &= C_{F,t} + I_t + G_t && (36) \\ I_t &= K_{t+1} - (1-\delta)K_t && (37) \\ K_t &= K_{F,t} + K_{I,t} && (38) \\ Y_{I,t} &= C_{I,t} && (39) \end{aligned}$$

with aggregate production functions (34).

Given government spending G_t, technology $A_{i,t}$, the nominal interest rate $R_{n,t}$, the real wage norm RW_t and choice of numeraire, the above system defines a general equilibrium in C_t, P_t, $P_{i,t}$, $P^W_{i,t}$, $C_{i,t}$, $h_{F,t}$, $h_{I,t}$, $W_{F,t}$, $W_{I,t}$, $n_{i,t}$, $Y_{i,t} = (1 - c_i)Y^W_{i,t}$ and $\hat{P}_{i,t}$ for $i = I, F$.

3.5 Monetary Policy and Government Budget Constraint

Monetary policy in conducted in terms of the nominal interest rate $R_{n,t}$ set at the beginning of period t. The expected real interest rate over the interval $[t, t+1]$ is given by

$$E_t[1 + R_{t+1}] = E_t \left[(1 + R_{n,t}) \frac{P_t}{P_{t+1}} \right] \tag{40}$$

In what follows we consider interest rate policy in the form of ad hoc Taylor-type rules, optimized Taylor rules, optimal commitment rules and finally discretionary policy.

Fiscal policy assumes a balanced budget constraint in which and employment tax on only formal firms, τ_t, finances government spending. This takes the form

$$P_{F,t}G_t = (n_{F,t}\tau_{F,t} + n_{I,t}\tau_{I,t})P_t h_t \tag{41}$$

noting that government services are provided out of formal output. We assume a *tax rule*

$$\tau_{I,t} = k\tau_{F,t}\,;\ \ k \in [0, 1] \tag{42}$$

allowing for the possibility that *some* tax can be collected in the informal economy.

3.6 Functional Forms

We choose a Cobb-Douglas production function, AR(1) processes for government spending and labour-augmenting productivity (LAP), and a utility function consistent with balanced growth:

$$
\begin{aligned}
F(A_{i,t}, N_{i,t}) &= (A_{i,t}N_{i,t})^{\alpha_i} K_{i,t}^{1-\alpha_i} && (43)\\
\log A_{i,t} - \log \bar{A}_{i,t} &= \rho_{A_i}(A_{i,t-1} - \bar{A}_{i,t-1}) + \epsilon_{A_i,t} && (44)\\
\log G_t - \log \bar{G}_t &= \rho_G(G_{t-1} - \bar{G}_{t-1}) + \epsilon_{G,t} && (45)\\
U_t(C_t, L_{i,t}) &= \frac{[C_t^{1-\varrho} L_{i,t}^{\varrho}]^{1-\sigma} - 1}{1-\sigma}\,; \quad \sigma > 1 \\
&= (1-\varrho)\log C_t + \varrho \log L_{i,t}\,; \quad \sigma = 1 && (46)\\
\log\left[\frac{\bar{A}_{i,t}}{\bar{A}_{i,t-1}}\right] &= \log\left[\frac{\bar{G}_t}{\bar{G}_{t-1}}\right] = 1 + g && (47)
\end{aligned}
$$

where $\epsilon_{A_i,t}, \epsilon_{G_i,t}, \sim ID$ with zero mean. The choice of utility function in (46) is chosen to be consistent with a steady state balanced growth path (henceforth BGP) where LAP $\bar{A}_t$ and $\bar{G}_t$ are time-varying. As pointed out in Barro and Sala-i-Martin (2004), chapter 9, this requires a careful choice of the form of the utility as a function of consumption and labour effort. It is achieved by a utility function which is non-separable in consumption and leisure unless $\sigma = 1$. A utility function of the form (46) achieves this. The marginal utilities are then then given by

$$
\begin{aligned}
\Lambda_{C,t} &= (1-\varrho)C_t^{(1-\varrho)(1-\sigma)-1}(n_{F,t}L_{F,t}^{\varrho(1-\sigma)} + (1-n_{I,t})L_{I,t}^{\varrho(1-\sigma)}) && (48)\\
U_{L_F,t} &= \varrho C_t^{(1-\varrho)(1-\sigma)} L_{F,t}^{\varrho(1-\sigma)-1} && (49)\\
U_{L_I,t} &= \varrho C_t^{(1-\varrho)(1-\sigma)} L_{I,t}^{\varrho(1-\sigma)-1} && (50)
\end{aligned}
$$

3.7 The Steady State and Model Calibration

The zero inflation balanced growth steady state of the model economy is given by

$$
\frac{\bar{\Lambda}_{C,t+1}}{\bar{\Lambda}_{C,t}} \equiv 1 + g_{\Lambda_C} = \left[\frac{\bar{C}_{t+1}}{\bar{C}_t}\right]^{(1-\varrho)(1-\sigma)-1)} = (1+g)^{((1-\varrho)(1-\sigma)-1)} \tag{51}
$$

using (48). Thus from (5)

$$
1 + R_n = 1 + R = \frac{(1+g)^{1+(\sigma-1)(1-\varrho)}}{\beta} \tag{52}
$$

$$
\begin{aligned}
n_{I,t} + n_{F,t} &= 1 & (53)\\
P &= \left[\mathrm{w}(P_F)^{1-\mu} + (1-\mathrm{w})(P_I)^{1-\mu}\right]^{\frac{1}{1-\mu}} & (54)\\
\bar{Y}_{i,t} &= (1-c_i)(n_i h_i \bar{A}_{i,t})^{\alpha_i} \bar{K}_{I,t}^{1-\alpha_i}\,;\, i = F, I & (55)\\
\frac{\varrho \bar{C}_t}{(1-\varrho)(1-h_i)} &= \bar{W}_{i,t}\,;\, i = F, I & (56)\\
\frac{\alpha_i P_I^W \bar{Y}_{i,t}^W}{P n_i h_i} &= \bar{W}_{i,t} + \bar{\tau}_{i,t}\,;\, i = F, I & (57)\\
\frac{\bar{W}_{F,t}}{P_t} &= R\bar{W}_t & (58)\\
\frac{\bar{K}_{F,t}}{\bar{Y}_{F,t}^W} &= \frac{1-\alpha_F}{R+\delta} & (59)\\
\frac{P_F^W \bar{K}_{I,t}}{P_I^W \bar{Y}_{I,t}^W} &= \frac{1-\alpha_I}{R+\delta} & (60)\\
\bar{I}_t &= (\delta + g)(\bar{K}_{I,t} + \bar{K}_{F,t}) & (61)
\end{aligned}
$$

$$
\begin{aligned}
\bar{Y}_{I,t} &= \bar{C}_{I,t} = (1-\mathrm{w})\left(\frac{P_I}{P}\right)^{-\mu} \bar{C}_t & (62)\\
\bar{Y}_{F,t} &= \bar{C}_{F,t} + \bar{G}_t = \mathrm{w}\left(\frac{P_F}{P}\right)^{-\mu} \bar{C}_t + \bar{I}_t + \bar{G}_t & (63)\\
\frac{P_F}{P}\bar{G}_t &= (n_F h_F \bar{\tau}_{F,t} + n_I h_I \bar{\tau}_{I,t}) & (64)\\
\bar{\tau}_{i,t} &= \tau_i \bar{W}_{i,t}\,;\, i = F, I & (65)\\
\bar{\tau}_{I,t} &= k\bar{\tau}_{F,t} & (66)\\
P_i &= \frac{1}{1-\frac{1}{\zeta_i}} P_i^W & (67)
\end{aligned}
$$

where consumption, technical LAP, the real wage and tax rates, and government spending (all indicated by $\bar{X}_t$) are growing at a common growth rate. We impose a free entry condition on retail firms in this steady state which drives monopolistic profits to zero. This implies that costs of converting wholesale to retail goods are given by

$$
c_i = 1/\zeta_i \qquad (68)
$$

which implies that:

$$
P_i \bar{Y}_{i,t} = P_i^W \bar{Y}_{i,t}^W\,;\, i = F, I \qquad (69)
$$

Given exogenous trends for $\bar{A}_{i,t}$ and $\bar{G}_t$, the tax rates and RW_t, equations (52)–(67) give 21 relationships in 22 variables R, P, P_F, P_I, P_F^W, P_I^W, $\bar{C}_t$, $\bar{C}_{F,t}$, $\bar{C}_{I,t}$, $\bar{Y}_{F,t}$, $\bar{Y}_{I,t}$,

$\bar{W}_{I,t}$, $\bar{W}_{F,t}$, n_I, n_F, h_I, h_F, $\bar{I}$, $\bar{K}_F\, \bar{K}_I$, $\bar{\tau}_{F,t}$, $\bar{\tau}_{I,t}$. One of the prices (it is convenient to choose P) can be chosen as the numeraire, so the system is determinate.

Turning to the calibration, the idea is to assume an observed baseline steady state equilibrium in the presence of some observed policy. We then use this observed equilibrium to solve for model parameters consistent with this observation For this baseline and for the purpose of calibration only, it is convenient to choose units of wholesale output such that their prices are unitary; i.e., $P_F^W = P_I^W = 1$. Then from (67)

$$P = \left[\frac{\mathrm{w}}{\left(1 - \frac{1}{\zeta_F}\right)^{1-\mu}} + \frac{(1-\mathrm{w})}{\left(1 - \frac{1}{\zeta_I}\right)^{1-\mu}} \right]^{\frac{1}{1-\mu}} \tag{70}$$

Thus if $\zeta_F = \zeta_I = \zeta$, $P = 1$, but we retain the possibility that price stickiness can be different in the two sectors. Similarly, we can choose units of labour supply h_I, h_F so that $A_I = 1$.

We now calibrate the parameters $\frac{\bar{A}_{F,t}}{\bar{A}_{I,t}}$, ϱ, w, β given observations of n_F, $\frac{\bar{Y}_{F,t}}{\bar{Y}_{I,t}} \equiv rel$, h_F, $\frac{\bar{W}_F}{\bar{W}_I} \equiv 1 + rw$, $ws_i \equiv \frac{P\bar{W}_{i,t} n_i h_i}{P_i^W \bar{Y}_{i,t}^W}$ $i = I, F$ R and $\frac{\bar{G}_t}{\bar{Y}_{F,t}} \equiv g_{yF}$. We also use estimates of σ and δ from micro-econometric studies. Denote observations by n_F^{obs} etc. With these observations and the steady state of the model we can deduce the unobserved variables in the steady state and the parameter values as follows:

From (56) we have

$$\frac{1 - h_I}{1 - h_F^{obs}} = \frac{\bar{W}_F}{\bar{W}_I} = 1 + rw^{obs} \tag{71}$$

which determines h_I. Hence we can obtain

$$ws_I = \frac{ws_F^{obs}(1 - n_F^{obs}) h_I\, rel^{obs}}{(1 + rw^{obs}) n_F^{obs} h_F^{obs}} \tag{72}$$

From the government budget constraint (64) in our baseline both formal are informal sectors are taxed ($k \in [0, 1]$) giving

$$g_{yF} = \tau_F (1 + k) ws_F^{obs} \tag{73}$$

which determines τ_F and $\tau_I = k\tau_F$. Then from (57)

$$\alpha_F = ws_F^{obs}(1 + \tau_F) \tag{74}$$

$$\alpha_I = ws_I \tag{75}$$

determining α_i, $i = I, F$.

Now write the production functions (55) as

$$\bar{Y}^W_{i,t} = A_{i,t} n_i h_i \left(KY_i\right)^{\frac{1-\alpha_i}{\alpha_i}} \quad i = I, F \tag{76}$$

where KY_i is the capital-labour ratio in sector i. From (59) and (60) and using $P^W_F = P^W_I = 1$ in the baseline steady state we have

$$KY_i = \frac{1-\alpha_i}{R^{obs} + \delta}\,; \, i = I, F \tag{77}$$

and (76) we have

$$\frac{\bar{Y}_{F,t}}{\bar{Y}_{I,t}} = rel^{obs} = \frac{\bar{A}_{F,t}}{\bar{A}_{F,t}} \frac{n^{obs}_F}{1-n^{obs}_F} \frac{h^{obs}_F}{h_I} \frac{KY_F^{\frac{1-\alpha_F}{\alpha_F}}}{KY_I^{\frac{1-\alpha_I}{\alpha_I}}} \tag{78}$$

from which $\frac{\bar{A}_{F,t}}{\bar{A}_{I,t}}$ is obtained.

To obtain w use (62) and (63) to give

$$\frac{w}{1-w} = \frac{\bar{Y}_{F,t}(1 - i_{yF} - g_{yF})}{\bar{Y}_{I,t}} = rel^{obs} c_{yF} \tag{79}$$

where

$$i_{yF} \equiv \frac{\bar{I}_t}{\bar{Y}_t} = \frac{(\delta+g)(\bar{K}_{I,t} + \bar{K}_{F,t})}{(1-c_F)\bar{Y}^W_{F,t}} = \frac{(\delta+g)\left(\frac{KY_I}{\hat{rel}} + KY_F\right)}{(1-c_F)} \tag{80}$$

$$c_{yF} = 1 - i_{yF} - g^{obs}_{yF} \tag{81}$$

From (79), (80) and (81) we now can determine w.

Finally from (56) and (62) we have

$$\frac{\varrho}{(1-\varrho)(1-h_I)} = (1-w)\frac{\bar{W}_{I,t}}{\bar{Y}_{I,t}} = \frac{(1-w)ws_I}{(1-n^{obs}_F)h_I} \tag{82}$$

from which ϱ is obtained. This completes the calibration; observations, imposed and calibrated parameters are summarized in Table 1.

In Table 2 the full steady-state benchmark equilibrium with no taxation in the in the informal sector used for the calibration is compared with a new steady state in which both sectors have the same tax rate. In this way, proceeding from $k = 1$ back to $k = 0$ we can show how the incentive to avoid taxation drives formalization. Thus we see in the tax-smoothing case a larger formal sector ($n_F = 0.33$ as opposed to $n_F = 0.25$ in the baseline case) and a lower relative price in the formal sector (because output is higher). Relative wages rise in the formal sector with the increase in demand for labour in that sector. The fall in the price of capital in the informal sector brings about a higher capital-output ratio and overall investment rises in the economy.

All this is with the wage mark-up in the formal sector $rw = 0.5$, our measure of wage stickiness. Figure 2 shows this process of informalization for different degrees of wage rigidity and illustrates how an increase in this friction also drives down participation in the formal sector. For example, with $k = 0$ and no friction the size of the formal sector is close to $n_F = 0.55$. When $rw = 0.75$, this falls to under $n_F = 0.2$.

Figure 3 shows the welfare effects on a representative household as the tax burden is smoothed over the two sectors. As k approaches unity the utility becomes very flat and close to the optimum. We can work out the equivalent permanent increase in consumption implied by this optimum by first computing the increase from a 1% consumption change at any point on the balanced growth trend as $n_F U(1.01 \times \bar{C}_t, L_F) + (1 - n_F)U(\bar{C}_t, L_I)$ at some time $t = 0$ say. In our best steady state equilibrium for $rw = 0.5$ at $k = 1$, this works out as 0.0059, so any increase in welfare $D\Lambda$ implies a consumption equivalent $c_e = \frac{D\Lambda}{0.0059}\%$ as calculated in Table 1.

Imposed Parameters	Value
δ	0.025
σ	2.0
$\xi_F = \xi_I$	0.75
$\zeta_F = \zeta_I$	7.0
μ	1.5
$\rho_{aF} = \rho_{aI} = \rho_g = \rho_{uI} = \rho_{uF}$	0.7
sd(ε_{aF})=sd(ε_{aI})= sd(ε_g)=sd(ε_{uF})= sd(ε_{uI})	2.0
Observed Equilibrium	**Value**
g^{obs}	0.01
n_F^{obs}	0.25
h_F^{obs}	0.5
rel^{obs}	2.0
ws_F^{obs}	0.5
rw^{obs}	0.5
g_{yF}^{obs}	0.2
R^{obs}	0.015
Calibrated Parameters	**Value**
α_I	0.80
α_F	0.60
β	0.998
w	0.37
ϱ	0.69

Table 1. Calibration

Variable	$k = 0$	$k = 1$
$\frac{PF}{P}$	1.00	0.8194
$\frac{PI}{P}$	1.00	1.1333
$\frac{W_F}{P}$	1.5381	1.6410
$\frac{W_I}{P}$	1.0254	1.0940
n_F	0.25	0.3264
h_F	0.5	0.4882
h_I	0.25	0.2323
rel	2.0	2.0363
R	0.015	0.015
τ_F	0.50	0.1520
τ_I	0.0	0.1520
KY_I	5.00	6.9158
KY_F	10.00	10.00
i_{yF}	0.51	0.5470
c_{yF}	0.29	0.2961
Λ	-1.8001	-1.7595 ($c_e = 0.81\%$)

Table 2. Steady State Equilibrium Values: $k = 0, 1$

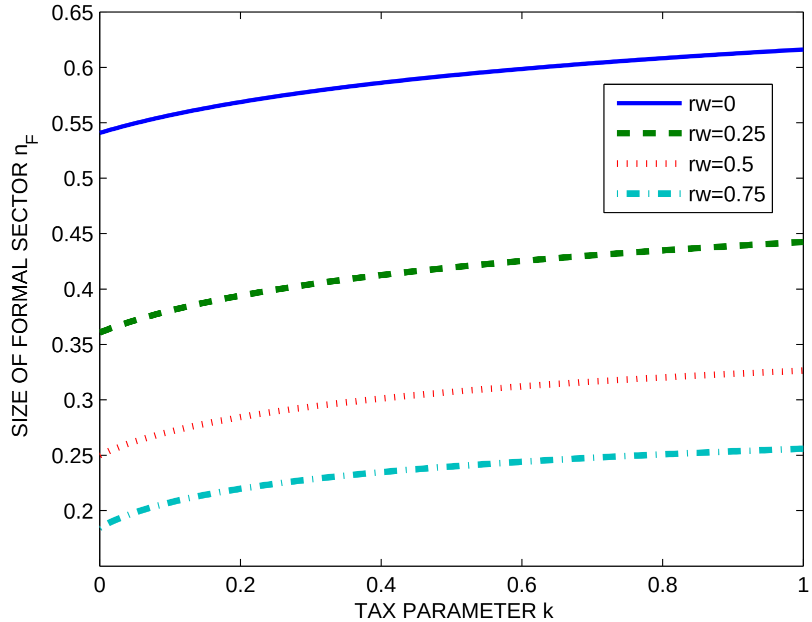

Figure 2: **The Size of Formal Sector and Tax Burden: k = Ratio of Informal-Formal Tax Rates. rw =wage mark-up in the formal sector.**

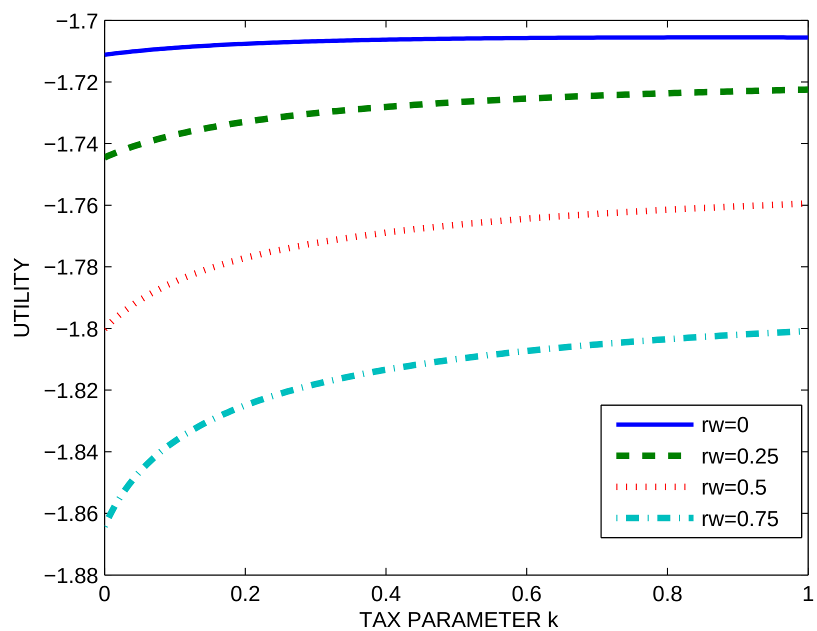

Figure 3: **Welfare and Tax Burden: k = Ratio of Informal-Formal Tax Rates. rw =wage mark-up in the formal sector.**

4 Optimal Policy and Optimized Rules

We adopt a linear-quadratic framework for the optimization problem facing the monetary authority. This is particularly convenient as we can then summarize outcomes in terms of unconditional (asymptotic) variances of macroeconomic variables and the local stability and determinacy of particular rules. The framework also proves useful for addressing the issue of the zero lower bound on the nominal interest rate.

In our model there are three distortions that result in the steady state output being below the social optimum: namely, from monopolistic competition, from distortionary taxes and from the non-market clearing wage norm. We assume that these distortions are small in the steady state and following Woodford (2003), we can adopt a 'small distortions' quadratic approximation to the household's single period utility which is accurate in the vicinity of our zero-inflation steady state.

To formulate this quadratic approximation first consider the simpler case without capital and with leisure constrained to be the same in both informal and formal sectors. Then we simply approximate the utility function $U_t = U(C_t, L_t)$ in consumption, C_t and leisure $L_t = 1 - h_t$ we start with the Taylor Series expansion about the BGP steady state[7]

$$U_t = U + U_C C c_t + \frac{1}{2} U_{CC} C^2 c_t^2 + U_L L l_t + \frac{1}{2} U_{LL} L^2 l_t^2 + \text{higher order terms} \tag{83}$$

Next we write $c_t = \mathrm{w} c_{F,t} + (1-\mathrm{w}) c_{I,t}$, $l_t = -\frac{h}{1-h}\hat{h}$ and use the linearized resource constraints

$$y_{F,t} = a_{F,t} + \alpha_F(\hat{n}_{F,t} + \hat{h}_t) - d_{F,t} = (1 - g_{Fy}) c_{F,t} + g_{Fy} g_t \tag{84}$$

$$y_{I,t} = a_{I,t} + \alpha_F(\hat{n}_{I,t} + \hat{h}_t) - d_{I,t} = c_{I,t} \tag{85}$$

where

$$d_{i,t} = \log\left[\frac{\Delta_{i,t}}{\Delta_i}\right] ; i = I, F \tag{86}$$

and $\Delta_{i,t}$ is the price dispersion effect given by (35). By standard results (see, for example, Gali (2008), p88) $d_{i,t}$ is a *second order* term given by

$$d_{i,t} = \frac{\zeta_i(\alpha_i + (1-\alpha_i)\zeta_i)}{2\alpha_i} \mathrm{var}(p_{i,t}(j)) \, ; i = I, F \tag{87}$$

[7]The BGP is time-varying but here we drop the bar and time-script in $\bar{U}_t, \bar{C}_t$ etc.

and

$$\sum_{t=0}^{\infty}\beta^t \text{var}(p_{i,t}(j)) = \frac{\xi_i}{(1-\beta\xi_i)(1-\xi_i)}\sum_{t=0}^{\infty}\beta^t\pi_{i,t}^2\,; i = I, F \tag{88}$$

Then using the linearized resource constraints and the properties of efficiency in the steady state: $\frac{U_L}{U_C} = F_{N_F} = F_{N_I}$ the first order terms in c_t and l_t disappear in (83) and we are left the quadratic approximation to the utility function

$$U_t = U + U_C C\left[-\frac{\text{w}}{(1-g_{Fy})}d_{F,t} - (1-\text{w})d_{I,t}\right] + \frac{1}{2}U_{CC}C^2c_t^2 + U_L L l_t + \frac{1}{2}U_{LL}L^2 l_t^2 + \text{higher order terms} \tag{89}$$

Finally using the results (86)–(89) we can write the quadratic form of the intertemporal expected welfare loss at time $t = 0$ as

$$\Omega_0 = \frac{1}{2}E_t\left[\sum_{t=0}^{\infty}\beta^t[w_c c_t^2 + w_h\hat{h}^2 + w_{\pi F}\pi_{F,t}^2 + w_{\pi I}\pi_{I,t}^2]\right] \tag{90}$$

where for our choice of utility function (46)

$$w_c = -\frac{U_{CC}C}{U_C} = 1 + (\sigma-1)(1-\varrho) \tag{91}$$

$$w_h = -\frac{U_{LL}h^2}{U_C C} = \frac{(1+\varrho(\sigma-1))h^2}{(1-\varrho)(1-h)^2} \tag{92}$$

$$w_{\pi F} = \text{w}\frac{\zeta_F(\alpha_F + (1-\alpha_F)\zeta_F)}{c_{Fy}\alpha_F\lambda_F} \tag{93}$$

$$w_{\pi I} = \frac{(1-\text{w})\zeta_I(\alpha_I + (1-\alpha_I)\zeta_I)}{\alpha_I\lambda_I} \tag{94}$$

$$\lambda_i = \frac{\xi_i}{(1-\beta\xi_i)(1-\xi_i)}\,; i = F, I \tag{95}$$

For the actual model with capital and different choices of work effort in the two sectors we use a modified version of this approximation:

$$\Omega_0 = \frac{1}{2}E_t\left[\sum_{t=0}^{\infty}\beta^t[w_c c_t^2 + w_{hI}\hat{h}_I^2 + w_{hF}\hat{h}_F^2 + w_{\pi F}\pi_{F,t}^2 + w_{\pi I}\pi_{I,t}^2]\right] \tag{96}$$

where now

$$w_h = -\frac{U_{LL}h^2}{U_C C} = \frac{(1+\varrho(\sigma-1))h^2}{(1-\varrho)(1-h)^2}\,;\ h = h_I, h_F \tag{97}$$

To work out the welfare in terms of a consumption equivalent percentage increase, expanding $U(C, L)$ as a Taylor series, a 1% permanent increase in consumption of 1 per cent yields a first-order welfare increase $U_C C \times 0.01$. Since standard deviations are expressed in terms of percentages, the welfare loss terms which are proportional to the covariance matrix (and pre-multiplied by 1/2) are of order 10^{-4}. The losses reported in the paper are scaled by a factor $1-\beta$. Letting $\Delta\Omega$ be these losses relative to the optimal policy, then $c_e = \Delta\Omega \times 0.01\%$.

We can modify welfare criterion so as to approximately impose an interest rate zero lower bound (ZLB) so that this event hardly ever occurs. Our quadratic approximation to the single-period loss function can be written as $L_t = \mathrm{y}_t' Q \mathrm{y}_t$ where $\mathrm{y}_t' = [\mathrm{z}_t', \mathrm{x}_t']'$ and Q is a symmetric matrix. As in Woodford (2003), chapter 6, the ZLB constraint is implemented by modifying the single period welfare loss to $L_t + w_r r_{n,t}^2$. Then following Levine *et al.* (2008b), the policymaker's optimization problem is to choose w_r and the unconditional distribution for $r_{n,t}$ (characterized by the steady state variance) shifted to the right about a new non-zero steady state inflation rate and a higher nominal interest rate, such that the probability, p, of the interest rate hitting the lower bound is very low. This is implemented by calibrating the weight w_r *for each of our policy rules* so that $z_0(p)\sigma_r < R_n$ where $z_0(p)$ is the critical value of a standard normally distributed variable Z such that prob $(Z \leq z_0) = p$, $R_n = \frac{1}{\beta(1+g_{u_c})} - 1 + \pi^*$ is the steady state nominal interest rate, $\sigma_r^2 = \mathrm{var}(r_n)$ is the unconditional variance and π^* is the new steady state inflation rate. Given σ_r the steady state positive inflation rate that will ensure $r_{n,t} \geq 0$ with probability $1-p$ is given by[8]

$$\pi^* = \max[z_0(p)\sigma_r - \left(\frac{1}{\beta(1+g_{u_c})} - 1\right) \times 100, 0] \tag{98}$$

In our linear-quadratic framework we can write the intertemporal expected welfare loss at time $t=0$ as the sum of stochastic and deterministic components, $\Omega_0 = \tilde{\Omega}_0 + \bar{\Omega}_0$. Note that $\bar{\Omega}_0$ incorporates in principle the new steady state values of all the variables; however the NK Phillips curve being almost vertical, the main extra term comes from the $\pi F, t^2$ and $\pi I, t^2$ terms in (90). By increasing w_r we can lower σ_r thereby decreasing π^* and reducing the deterministic component, but at the expense of increasing the stochastic component of the welfare loss. By exploiting this trade-off, we then arrive at the optimal policy that, in the vicinity of the steady state, imposes the ZLB constraint, $r_t \geq 0$ with probability $1-p$.

We now return to *symmetrical* and *asymmetrical* interest rate Taylor rule that responds

[8]If the inefficiency of the steady-state output is negligible, then $\pi^* \geq 0$ is a credible new steady state inflation rate. Note that in our LQ framework, the zero interest rate bound is very occasionally hit in which case the interest rate is allowed to become negative.

to deviations of inflation and the output gap in both formal and informal sectors; but now we allow for a degree of interest rate smoothing. We write the rules as:

$$r_{n,t} = \rho r_{n,t-1} + \theta_\pi \pi_t + \theta_{Fy}(y_{F,t} - y^*_{F,t}) + \theta_{Iy}(y_{I,t} - y^*_{I,t})] \quad (99)$$

$$r_{n,t} = \rho r_{n,t-1} + \theta_\pi \pi_{F,t} + \theta_{Fy}(y_{F,t} - y^*_{F,t}) \quad (100)$$

and we compute optimal parameter values that optimize Ω_0. The results are displayed in Tables 3. Again the consumption equivalent changes in utility are measured relative to the best outcome which is the optimal policy with the formal sector at its higher value. Thus $c_e = 0$ in this case.

n_F	Rule	w_r	$[\rho,\ \theta_{\pi F},\ \theta_{\pi I},\ \theta_{yF},\ \theta_{yI}]$	Ω_0	σ_r^2	π^*	Pr(ZLB)	c_e
0.25	Sym TR	0	[0.98, 0.00, 0.05, 0.00, 0.00]	30.96	0.029	0	0.000	0.20
0.25	Asy TR	0	[1.00, 0.01, 0, 0.02, 0]	31.61	0.011	0	0.000	0.20
0.25	Optimal	0	complex	25.08	0.095	0	0.000	0.13
0.36	Sym TR	0	[1.00, 0.02, 1.38, 0.06, 0.05]	39.31	0.055	0	0.000	0.27
0.36	Asym TR	0	[0.91, 0.30, 0, 0.02, 0]	46.30	0.110	0	0.000	0.34
0.36	Optimal	0	complex	12.00	0.037	0	0.000	0

Table 3. Optimal Rules: No ZLB Imposed.

A number of points should be highlighted. First ZLB since the interest rate volatilities are so low, ZLB considerations are irrelevant for this model and calibration. Second in the baseline $n_F = 0.25$ equilibrium, a simple optimized Taylor rule gets a consumption equivalent $c_e = 0.20 - 0.13 = 0.07\%$ of mimicking the optimal rule and does so with an near integral rule that responds strongly to inflation in the informal sector almost entirely, but hardly at all to the output gaps. The latter is a familiar finding in the literature for one-sector models. Third, in the baseline equilibrium the performance of the asymmetrical rule deteriorate only slightly and the optimized feedback on formal inflation is more muted. There is now less of a role for monetary stabilization. Finally in the larger informal sector equilibrium the stabilization benefits of tax smoothing outweigh any cost of producing more in a sector with labour market frictions, but only as long as the fully optimal rule is implementable. In reality this is implausible as the optimal rule is exceedingly complex and relies on the ability to feed back on shocks. So the Taylor rules are more interesting and here we see a stabilization cost on a larger formal sector wih wage rigidities now outweighing tax benefits. Moreover there is now a significant (but not large) cost of an asymmetric rule of $c_e = 0.07\%$. Table 4 summarizes this cost-benefit analysis bringing the earlier steady state and stabilization results together.

Source of Cost	Consumption Equivalent Cost c_e (%)
Tax Smoothing at Steady State	0.81
Stabilization Cost: Optimal Rule	0.13
Stabilization Cost: Symmetric Taylor Rule	-0.07
Stabilization Cost: Asymmetric Taylor Rule	-0.14

Table 4. The Cost (and Benefit) of Informalization.

To assess this table it is important to stress that stabilization depend on the calibrated volatilities of the shocks. We assumed a standard deviation of 2% for all shocks, which is a plausible figure for emerging economies and in line with DSGE estimation. Of course volatilities for some countries could be higher, so let the standard deviation be scaled by factor $k \geq 1$. Then we can see from Table 4 that stabilization gains from informalization with an asymmetric Taylor rule will outweigh the tax smoothing at the steady state iff $0.14k^2 > 0.81$ which occurs iff $k > 2.41$ and the standard deviation is at the (enormous) value of 4.82%. We must conclude that in our model and with our calibration, any net gains from informalization are not possible.

5 Conclusions

The main conclusion of our paper is that unless shocks have volatilities at implausible levels, the cost of informalization in terms of losing out on tax smoothing by far outweigh any stabilization benefit from increased wage flexibility. Although the latter also disappear if the optimal rule is available, it emerges if rules are realistically constrained to be of the simple Taylor-form.

A number of caveats should be mentioned. First in the model we ignore frictions arising from investment costs. This means that capital adjusts immediately to shocks and can therefore compensate to an extent for other frictions. It follows that the business cycle costs in the model are underestimated; with this feature the interest rate volatility is low and the ZLB is not a problem (unlike, for example, Levine *et al.* (2008b) in a model with investment frictions). Another limitation of this study is the use of the small-distortions LQ approximation for the utility. Although this is common in the optimal monetary policy literature, it is a convenient short-cut that may have important consequences given that the distortions in both our steady states are likely to be large without the fiction of non-distortionary taxes available to the policymakers.[9] Finally it would be desirable to estimate the model by Bayesian methods as is now commonplace in the literature. For advanced economies the informal sector would become the hidden economy leading to the need to properly into account the lack of observability of this sector in solving for the rational expectations equilibrium and the estimation. This is not done in this paper, nor indeed in the DSGE literature as a whole.[10] These three caveats suggest future important directions for research.

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A Linearization

Define lower case variables $x_t = \log \frac{X_t}{\bar{X}_t}$ if X_t has a long-run trend or $x_t = \log \frac{X_t}{X}$ otherwise where X is the steady state value of a non-trended variable. For variables $n_{F,t}$, $n_{I,t}$ and h_t define $\hat{x}_t = \log \frac{x_t}{x}$; $r_{n,t} \equiv \log\left(\frac{1+R_{n,t}}{1+R_n}\right)$; $\pi_{i,t} \equiv \log\left(\frac{1+\Pi_{i,t}}{1+\Pi_i}\right)$, $i = I, F$ are log-linear *gross* interest and inflation rates.

Our linearized model about the BGP zero-inflation steady state then takes the state-space form form

$$a_{F,t+1} = \rho_{aF} a_{F,t} + \varepsilon_{aF,t+1} \tag{A.1}$$

$$a_{I,t+1} = \rho_{aI} a_{I,t} + \varepsilon_{aI,t+1} \tag{A.2}$$

$$g_{t+1} = \rho_g g_t + \varepsilon_{g,t+1} \tag{A.3}$$

$$u_{F,t+1} = \rho_{uF} u_{F,t} + \varepsilon_{uF,t+1} \tag{A.4}$$

$$u_{I,t+1} = \rho_{uI} u_{I,t} + \varepsilon_{uI,t+1} \tag{A.5}$$

$$\tau_t = \tau_{t-1} + \pi_{I,t} - \pi_{F,t} \tag{A.6}$$

$$k_t = \frac{1-\delta}{1+g} + \frac{\delta+g}{1+g} i_t \tag{A.7}$$

$$E_t[\lambda_{C,t+1}] = \lambda_{C,t} - E_t[r_t] \tag{A.8}$$

$$\beta E_t[\pi_{F,t+1}] = \pi_{F,t} - \lambda_F (mc_{F,t} + u_{F,t}) \tag{A.9}$$

$$\beta E_t[\pi_{I,t+1}] = \pi_{I,t} - \lambda_I (mc_{I,t} + u_{I,t}) \tag{A.10}$$

with outputs defined by

$$E_t[r_t] = r_{n,t} - E_t[\pi_{t+1}] \tag{A.11}$$

$$E_t[\pi_{t+1}] = \mathrm{w} E_t[\pi_{F,t+1}] + (1-\mathrm{w}) E_t[\pi_{I,t+1}] \tag{A.12}$$

$$\pi_t = \mathrm{w}\pi_{F,t} + (1-\mathrm{w})\pi_{I,t} \tag{A.13}$$

$$c_t : \lambda_{C,t} = -(1 + (\sigma-1)(1-\varrho) c_t + \frac{n_F (L_F^{\varrho(1-\sigma)} - L_I^{\varrho(1-\sigma)}) \hat{n}_{F,t} + \varrho(\sigma-1)(n_F L_F^{\varrho(1-\sigma)} \ell_{F,t} + (1-n_F) L_I^{\varrho(1-\sigma)} \ell_{I,t})}{n_F L_F^{\varrho(1-\sigma)} + (1-n_F) L_I^{\varrho(1-\sigma)}} \tag{A.14}$$

$$u_{L_I,t} = u_{C,t} + c_t + \frac{h_I}{1-h_I} \hat{h}_{I,t} \tag{A.15}$$

$$u_{L_F,t} = u_{C,t} + c_t + \frac{h_F}{1-h_F} \hat{h}_{F,t} \tag{A.16}$$

$$w_{I,t} - p_t = u_{L_I,t} - \lambda_{C,t} \tag{A.17}$$

$$
\begin{aligned}
\hat{h}_{F,t}:\ w_{F,t} - p_t &= u_{L_F,t} - \lambda_{C,t} && \text{(A.18)}\\
w_{F,t} - p_t &= \omega(w_{I,t} - p_t) && \text{(A.19)}\\
c_{F,t} &= c_t + \mu(1-\mathrm{w})\tau_t && \text{(A.20)}\\
c_{I,t} &= c_t - \mu \mathrm{w} \tau_t && \text{(A.21)}\\
\hat{n}_{F,t}:\ y_{F,t} &= a_{F,t} + \alpha_F(\hat{n}_{F,t} + \hat{h}_{F,t}) - (1-\alpha_F)k_{F,t} && \text{(A.22)}\\
\hat{h}_{I,t}:\ y_{I,t} &= a_{I,t} + \alpha_I(\hat{n}_{I,t} + \hat{h}_{I,t}) - (1-\alpha_I)k_{I,t} && \text{(A.23)}\\
\hat{n}_{I,t} &= -\frac{n_F}{n_I}\hat{n}_{F,t} && \text{(A.24)}\\
mc_{F,t} &= \frac{1}{1+\tau_F}(w_{F,t} - p_t) + \frac{\tau_F}{1+\tau_F}\hat{\tau}_{F,t} + (1-\mathrm{w})\tau_t - a_{F,t} \\
&+ (1-\alpha_F)(\hat{n}_{F,t} + \hat{h}_{F,t} - k_{F,t}) && \text{(A.25)}\\
mc_{I,t} &= \frac{1}{1+\tau_I}(w_{I,t} - p_t) + \frac{\tau_I}{1+\tau_I}\hat{\tau}_{I,t} - \mathrm{w}\tau_t - a_{I,t} \\
&+ (1-\alpha_I)(\hat{n}_{I,t} + \hat{h}_{I,t} - k_{I,t}) && \text{(A.26)}\\
y_{I,t} &= c_{I,t} && \text{(A.27)}\\
i_t:\ y_{F,t} &= c_{yF}c_{F,t} + i_{yF}i_t + g_{yF}\, g_t && \text{(A.28)}\\
g_t &= (1-\mathrm{w})\tau_t + \frac{n_F\tau_F}{n_F\tau_t + n_I\tau_I}(\hat{n}_{F,t} + \hat{h}_{F,t} + \hat{\tau}_{F,t}) + \frac{n_I\tau_I}{n_F\tau_t + n_I\tau_I}(\hat{n}_{I,t} + \hat{h}_{I,t} + \hat{\tau}_{I,t})\\
\hat{\tau}_{I,t} &= \hat{\tau}_{F,t} && \text{(A.29)}
\end{aligned}
$$

$$
\begin{aligned}
k_{I,t}:\ k_t &= \frac{\bar{K}_{F,t}}{\bar{K}_t}k_{F,t} + \frac{\bar{K}_{I,t}}{\bar{K}_t}k_{I,t} && \text{(A.30)}\\
y_{F,t}:\ y_{F,t} - k_{F,t} &= \frac{1+R}{R+\delta}r_t && \text{(A.31)}\\
k_{F,t}:\ mc_{F,t} &= mc_{I,t} + \tau_t + y_{I,t} - y_{F,t} + k_{F,t} - k_{I,t} && \text{(A.32)}
\end{aligned}
$$

where $\lambda_i \equiv \frac{(1-\beta\xi_i)(1-\xi_i)}{\xi_i}$, and $\tau_i \equiv \frac{\bar{\tau}_i}{W_t/P}$ $i = I, F$. Note that (A.14) defines c_t, (A.22) defines $\hat{n}_{F,t}$ and (A.23) defines $\hat{h}_t$. Let $\tau_I = (1-k)\tau_F$ where $k \in [0,1]$ to allow taxation to be enforced in the informal sector. Also (A.20) and (A.21) implies $c_t = \mathrm{w}c_{F,t} + (1-\mathrm{w})c_{I,t}$

The flexi-price 'natural rate' economy is found by putting $mc_{F,t} = mc_{I,t} = 0$ and making taxes non-distortionary.