AN UNOBSERVED COMPONENTS PHILLIPS CURVE FOR INDIA

12TH RESEARCH MEETING OF NIPFP-DEA RESEARCH PROGRAMME

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University of Oxford
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3. Evolution of the Phillips Curve
4. What is new?
5. Modeling Output Gap
6. Unobserved Components Phillips Curve
7. Results
8. Conclusion
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WHAT IS THE PHILLIPS CURVE (PC)

- Relationship between Inflation and output gap

<table>
<thead>
<tr>
<th>Output Gap: Difference between actual and potential GDP</th>
</tr>
</thead>
<tbody>
<tr>
<td>If <strong>positive</strong>, it is called an inflationary gap. Indicates the growth of aggregate demand is outpacing the growth of aggregate supply—possibly creating inflation</td>
</tr>
</tbody>
</table>

- Output gap proxies demand pressures
- PC quantifies this relationship: **positive relationship**
# UC Phillips Curve for India

## What is the Phillips Curve?

## Motivation

## Evolution of the Phillips Curve

## What is new?

## Modeling Output Gap

## Unobserved Components Phillips Curve

## Results

## Conclusion
MOTIVATION

• Resurgence of PC literature:
  • World
    (JME 2005 special edition)
  • India
    • dismissal
    • to gradual acceptance
      [Paul 2009, Singh et. al. 2011, Kapur 2013]

• Informing monetary policy: agnostic → believer

• The main objective of the Reserve Bank of India (RBI), is “...formulation and implementation of monetary policy with the objectives of maintaining price stability...[and] to promote economic growth...”
  [Reserve Bank of India: Functions and Working]
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EVOLUTION OF THE PC I

- PC determines inflation
- Inflation = linear combination of persistence/expectations + demand + supply
- The great Bifurcation [Gordon 2011]

<table>
<thead>
<tr>
<th>Keynesian/ Inertial/ Backward-looking PC</th>
<th>New Keynesian Phillips Curve/Forward-looking PC</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi_t = \sum_{i=1}^{n} \alpha_i \pi_{t-i} + \beta (Y_t - Y_t^n) + \gamma z_t + \varepsilon_t$</td>
<td>$\pi_t = \alpha E_t \pi_{t+1} + \beta (Y_t - Y_t^n) + \varepsilon_t$</td>
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<tr>
<td>Persistence captured by past lags</td>
<td>Forward looking expectations</td>
</tr>
<tr>
<td>Supply shocks enter explicitly</td>
<td>Relegated to error term</td>
</tr>
</tbody>
</table>

$\pi_t$: inflation  
$Y_t$: GDP  
$Y_t^n$: potential GDP  
$z_t$: catch-all supply shock variable  
$\alpha, \beta, \gamma$: constants  
$E_t \pi_{t+1}$: expectations at time $t$ for inflation at time $t + 1$  
$\varepsilon_t$: White noise error term
• PC determines inflation
• Inflation = linear combination of persistence/expectations + demand + supply
• The great Bifurcation [Gordon 2011]

### Keynesian/ Inertial/ Backward-looking PC

\[ \pi_t = \sum_{i=1}^{n} \alpha_i \pi_{t-i} + \beta (Y_t - Y_t^n) + \gamma z_t + \varepsilon_t \]

- Persistence captured by past lags
- Supply shocks enter explicitly

### New Keynesian Phillips Curve/ Forward-looking PC

\[ \pi_t = \alpha E_t \pi_{t+1} + \beta (Y_t - Y_t^n) + \varepsilon_t \]

- Forward looking expectations
- Relegated to error term

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$E_t \pi_{t+1}$: expectations at time $t$ for inflation at time $t + 1$  
$\varepsilon_t$: White noise error term
• Empirically, NKPC is a failure [Gordon 2011]

• “Expectations are the *sine-qua-non* of Macroeconomics” [Harvey 2011]

• Hybrid New Keynesian Phillips curve [Gali and Gertler 1999]
  • Most of the fit comes from backward looking part! [Rudd and Whelan 2005 and 2007]
  • Large robust confidence intervals [Kleibergen and Mavroeidis 2009]
Simple modification of a backward-looking PC [Harvey 2011]

- lagged inflation is replaced by an **unobserved random walk** component: from ARDL to UC

  captures the **underlying level of inflation**

WHAT IS NEW II: WHY A UC MODEL?

- Allows flexible treatment of non-stationary time series

- Convenient trend-cycle decomposition

  - HP unsuitable for developing countries [Ozbek and Ozlle 2005]
  - HP’s end point bias [Mise et. al. 2005, Harvey and Trimbur 2008 and Harvey and Delle Monache 2009]
  - Ambiguity of HP’s smoothing parameter ($\lambda$)
• Use of quarterly GDP over annual GDP
  - Less use for policy
  - Structural breaks

• Use of GDP over convenient proxies like IIP
  - Captures only small part of economy
  - IIP vs. economy wide WPI is imprudent

• Sequential growth rates Q-o-Q over Y-o-Y figures

• Out of sample extrapolative tests
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A trend-cycle decomposition is set up

- $y_t$ is log (GDP), $\varepsilon_t$ is white noise, $\mu_t$ is an integrated random walk:

$$y_t = \mu_t + \psi_t + \gamma_t + \varepsilon_t \quad \varepsilon_t \sim NID(0, \sigma^2_\varepsilon) \quad \text{...(1)}$$

$$\mu_t = \mu_{t-1} + \beta_{t-1} \quad \text{...(2)}$$

$$\beta_t = \beta_{t-1} + \zeta_t \quad \zeta_t \sim NID(0, \sigma^2_\zeta) \quad \text{...(3)}$$

- $\gamma_t$ is the seasonal, $\psi_t$ is the cyclical and $\beta_t$ is the slope component and the normal white noise disturbances are independent of each other

- The smoothed components of the cycle give us the output gap
MODELING OUTPUT GAP II

- Seasonal and cyclical components are given by trigonometric formulations

1) SEASONAL COMPONENT: \( \gamma_t \) is given by:

\[
\gamma_t = \sum_{j=1}^{\left[\frac{s}{2}\right]} \gamma_{j,t}
\]

where each \( \gamma_{j,t} \) is generated by

\[
\begin{bmatrix}
\gamma_{j,t} \\
\gamma^*_{j,t}
\end{bmatrix} =
\begin{bmatrix}
\cos \lambda_j & \sin \lambda_j \\
-\sin \lambda_j & \cos \lambda_j
\end{bmatrix}
\begin{bmatrix}
\gamma_{j,t-1} \\
\gamma^*_{j,t-1}
\end{bmatrix} +
\begin{bmatrix}
\omega_{j,t} \\
\omega^*_{j,t}
\end{bmatrix}
\]

\[j = 1, \ldots, \left[\frac{s}{2}\right], \quad \omega_t \sim NID(0, \sigma^2_{\omega}), \quad \omega^*_t \sim NID(0, \sigma^2_{\omega}),\]

where \( \gamma_j = 2\pi j/s \) is the frequency, in radians, and the seasonal disturbances \( \omega_t \) and \( \omega^*_t \) are mutually and serially uncorrelated (with common variance, as above).
2) CYCLICAL COMPONENT: $\psi_t$ is specified by a ‘balanced cycle model’ of order $k = 2$ [Harvey and Trimbur 2003]

$$
\begin{bmatrix}
\psi_t^{(k)} \\
\psi_t^{*(k)}
\end{bmatrix} = \rho_\psi
\begin{bmatrix}
\cos \lambda & \sin \lambda \\
-sin \lambda & \cos \lambda
\end{bmatrix}
\begin{bmatrix}
\psi_{t-1}^{(k)} \\
\psi_{t-1}^{*(k)}
\end{bmatrix} +
\begin{bmatrix}
\psi_{t-1}^{(k-1)} \\
\psi_{t-1}^{*(k-1)}
\end{bmatrix}
\quad t = 1, \ldots, T,
$$

where $\lambda$ is frequency, in radians, in the range $0 \leq \lambda \leq \pi$; $\rho_\psi$ is the dampening factor, with $0 < \rho_\psi \leq 1$, are mutually and serially $\psi_t^{*(k)}$ is an auxiliary process.

- The period of the cycle is equal to $2\pi/\lambda_c$. For computing the output gap, this value is fixed at 20 quarters (five years), which is the length of a typical business cycle (Koopman et. al. 2007).
• The reduced form of the model is an Autoregressive Moving Average ARMA (2,1) process with complex roots for the autoregressive part.

• The model is assumed to be Gaussian and the hyperparameters \((\sigma_\varepsilon^2, \sigma_\eta^2, \sigma_\zeta^2, \sigma_\kappa^2, \lambda_c, \rho_\psi)\) are estimated using Maximum Likelihood.

• Diffuse initialisation is used to deal with the problem of unobserved state at the beginning of the time series.

• Estimates of trend, cyclical, seasonal, and irregular components are given by a smoothing algorithm in STAMP.
Log (GDP) and its decomposition into stochastic level and seasonal components.
MODELING OUTPUT GAP VI

OUTPUT GAP FOR INDIAN ECONOMY

- Asian crisis + political instability
- drought
- investment boom
- fiscal stimulus
- financial crisis
- current slowdown
MODELING OUTPUT GAP VII: RESIDUAL GRAPHICS

- **Density Plot:**
  - Standardised Residuals +/− 2SE
  - Density curve with bars

- **QQ Plot:**
  - Standardised Residuals ACF
  - QQ plot with line and points
  - Residuals × Normal

UC Phillips Curve for India
The diagnostic tests of state space time series models are based on properties of residuals:

1. Independence
2. Homoskedasticity
3. Normality

Is IRW a suitable model?

### TABLE 1: Diagnostic Tests for the GDP Model

<table>
<thead>
<tr>
<th>Diagnostic</th>
<th>Test</th>
<th>Value</th>
<th>Critical value (at10%)</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Independence**</td>
<td>Box-Ljung Q(q)</td>
<td>10.273</td>
<td>12.017</td>
<td>0.174</td>
</tr>
<tr>
<td></td>
<td>r(1)</td>
<td>0.100</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>r(q)</td>
<td>0.021</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Homoskedasticity**</td>
<td>1/H(20,20)</td>
<td>0.971</td>
<td>1.768</td>
<td>0.474</td>
</tr>
<tr>
<td>Normality**</td>
<td>Bowman-Shenton</td>
<td>0.202</td>
<td>4.605</td>
<td>0.904</td>
</tr>
</tbody>
</table>

** INDICATES THAT NULL HYPOTHESIS IS NOT REJECTED EVEN AT10% LEVEL OF SIGNIFICANCE
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MODEL A: A parsimonious ‘pure’ PC with output gap as the only explanatory variable.

\[ \pi_t = \mu_t + \psi_t + \beta_1 (OG_t) + \varepsilon_t, \quad \varepsilon_t \sim NID(0, \sigma_\varepsilon^2), \]
\[ \mu_t = \mu_{t-1} + \eta_t, \quad \eta_t \sim NID(0, \sigma_\eta^2) \]

\[ OG_t = Y_t - Y_t^n \]

- \( OG_t \) is the output gap.
- \( \mu_t \), measures trend inflation
- Modification of the backward looking PC
MODEL B: An ‘extended’ PC with controls for supply shocks built in (Kapur 2013).

A. Food: $Rain_t$
   - rainfall shortage during the month of July – the critical month for kharif sowing
   - $Rain_t$ measures per cent deviation in actual rain from normal rainfall during July.
   - Impacts inflation with a lag

B. Crude oil: $croil_t$ (IMF)

C. Non-oil commodity inflation: $nfuel_t$ (IMF)
D. Exchange rate: $NEER_t$

- Depreciation in the Rupee may increase price of imports without requisite demand pressure
- Nominal Effective Exchange Rate (NEER)
- Measures variation (y-o-y) in the 36-currency trade-weighted nominal effective exchange rate index of the Indian rupee complied by the RBI
- A decrease in NEER implies depreciation

- Thus the extended PC specification is:

$$\pi_t = \mu_t + \psi_t + \beta_1 OG_t + \beta_2 Rain_t + \beta_3 Croil_t$$

$$+ \beta_4 NFUEL_t + \beta_5 NEER_t + \varepsilon_t$$
### RESULTS I

Estimated Equations for Quarterly Changes in the WPI 1996:Q3 to 2013:Q1

Dependent variable: Annualized sequential growth rate of WPI

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>$OG_{t-1}$</td>
<td>1.71883 **</td>
<td>1.542**</td>
</tr>
<tr>
<td></td>
<td>(2.15845)</td>
<td>(2.047)</td>
</tr>
<tr>
<td>$Rain_{t-1}$</td>
<td>-</td>
<td>0.010 ***</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(2.678)</td>
</tr>
<tr>
<td>$Oil_{t-1}$</td>
<td>-</td>
<td>0.015*</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(1.916)</td>
</tr>
<tr>
<td>$Oil_{t-2}$</td>
<td>-</td>
<td>-0.013 **</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(-2.008)</td>
</tr>
<tr>
<td>$Nfuel_{t-1}$</td>
<td>-</td>
<td>0.045**</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(2.355)</td>
</tr>
<tr>
<td>$NEER$</td>
<td>-</td>
<td>-0.141***</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(-3.101)</td>
</tr>
</tbody>
</table>

$t$-values are reported in parentheses.

*, **, *** indicate significance at the 10%, 5%, and 1% level, respectively.
RESULTS II: COMPONENTS OF MODEL B

UC Phillips Curve for India
RESULTS III: RESIDUAL GRAPHICS OF MODEL B

- **Density**: Distribution of the data with a normal distribution curve.
- **QQ plot**: Quantile-quantile plot comparing the standardised residuals against the quantiles of the normal distribution.
- **Standardised Residuals +/- 2SE**: Graph showing the range of standardised residuals.
- **Standardised Residuals ACF**: Autocorrelation function of the standardised residuals.
## RESULTS IV: DIAGNOSTIC TESTS

### Diagnostic Statistics

<table>
<thead>
<tr>
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<tbody>
<tr>
<td>Std.error</td>
<td>2.886</td>
<td>2.219</td>
</tr>
<tr>
<td>PEV</td>
<td>8.328</td>
<td>4.927</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.465</td>
<td>0.709</td>
</tr>
<tr>
<td>Normality</td>
<td>0.789**</td>
<td>3.677**</td>
</tr>
<tr>
<td></td>
<td>(4.605)</td>
<td>(4.605)</td>
</tr>
<tr>
<td>H(h,h)</td>
<td>1.399**</td>
<td>1.0025**</td>
</tr>
<tr>
<td></td>
<td>(1.794)</td>
<td>(1.822)</td>
</tr>
<tr>
<td>DW</td>
<td>1.979</td>
<td>2.1128</td>
</tr>
<tr>
<td>r(1)</td>
<td>-0.028</td>
<td>-0.091204</td>
</tr>
<tr>
<td>r(q)</td>
<td>-0.087</td>
<td>0.0030983</td>
</tr>
<tr>
<td>Q(q,q-p)</td>
<td>6.629**</td>
<td>5.0586**</td>
</tr>
<tr>
<td></td>
<td>(10.646)</td>
<td>(10.646)</td>
</tr>
<tr>
<td>AIC</td>
<td>2.299</td>
<td>1.923</td>
</tr>
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* *INDICATES THAT NULL HYPOTHESIS IS NOT REJECTED AT 10% LEVEL OF SIGNIFICANCE. PARANTHESIS CONTAIN 10 PER CENT CRITICAL VALUES
RESULTS V: Out of sample multi-step ahead predictive test*

Model A

Actual WPI
Predicted WPI

Model B

Actual WPI
Predicted WPI
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The Phillips curve does exist for India!

The simple UC Phillips Curve is parsimonious and provides a good fit to the data.

Improvements in econometric implementation

UC models such as the UC Phillips curve can fit well in a good deal of Dynamic Stochastic General Equilibrium (DSGE) models

Disentangling and quantifying demand and supply pressures

Forecasting inflation
FURTHER RESEARCH

• Embodying unobserved components in a theoretical New-Keynesian Framework [Harvey 2011].

• Can the UC Phillips curve fit DSGE models for India?

• Improving over supply shock controls [Mankiw and Ball 1994, Gordon 1988].

• Model selection
The End
APPENDIX 1: DIAGNOSTIC TESTS

• The **Box-Ljung** test for serial correlation has been used to test serial correlation. It is based on residual autocorrelation of the first \( q \) lags. The test statistic is distributed as \( \chi^2 \) with \( (q - p - 1) \) degrees of freedom where \( p \) is the number of hyper-parameters to be estimated.

• For verifying homoskedasticity of the residuals, the **H statistic** is used which tests whether the variance of the residuals in the first third part of the series is equal to the variance of the residuals corresponding to the last third part of the series. This is tested employing a two tailed test against an F-distribution with \((h, h)\) degrees of freedom. \( h \) is the nearest integer to \((n - d)/3\), where \( n= \) number of observations, \( d= \) number of diffuse initial elements.

• Normality of residuals is tested against the **Bowman-Shenton** statistic which is based on the third and fourth moments of the residuals and has a \( \chi^2 \) distribution with 2 degrees of freedom.
APPENDIX 2: WORKING OF THE KALMAN FILTER

Figure 8.6. Illustration of computation of the filtered state for the local level model applied to Norwegian road traffic fatalities.