

Estimation of Value at Risk for the Indian capital market: Filtered Historical Simulation approach using GARCH model with suitable mean specification

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Abstract:

The paper estimate 1-day Value at Risk (VaR) taking into consideration the financial integration of Indian capital market (BSE-SENSEX and NSE-NIFTY) with other global indicators and its own volatility using daily returns covering the period January 2003 to December 2009. The paper specifies a generalized autoregressive conditional heteroscedasticity (GARCH) framework to model the phenomena of volatility clustering on returns and examines the usefulness of considering lag values of return on (S&P 500, INR-EURO & INR-USD exchange rate, Gold price) as proxies to global financial condition in the specification of the mean equation. In general VaR is calculated either based on Historical Simulation (HS) approach which imposes practically no structure on the distribution of returns except stationarity or using Monte Carlo simulation (MCS) approach which assumes parametric models for variance and subsequently large number of random numbers is drawn from this specific distribution to calculate the desired risk measure. Filtered Historical Simulation (FHS) approach attempts to combine the best of the model-based approach with the best of the model-free approaches in a very intuitive fashion. The paper estimates VaR of return in the Indian capital market based on two composite methods i.e. (a) using univariate GARCH model where in the mean equation we have used lag values of return on S&P 500, INR-EURO & INR-USD exchange rate and Gold price; and following FHS approach; (b) using ARMA for mean equation, GARCH for volatility and FHS for VaR estimation i.e. ARMA-GARCH-FHS. The performances of the VaR estimates from both the methods were compared and it was found that VaR of return in the Indian capital market estimated based on method (a) i.e. GARCH with suitable mean specification outperforms method (b) i.e. the ARMA-GARCH method.

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Estimation of Value at Risk for the Indian capital market: Filtered Historical Simulation approach using GARCH model with suitable mean specification

1. Introduction:

Globalisation and financial sector reforms in India lead to a greater integration of Indian stock market with the advanced economies also to the exchange rate movements. According to efficient market hypothesis, equity prices reflect all available relevant information fully and instantaneously. Fama (1970), describe three forms of market efficiency i.e. weak form, semi-strong form and strong form of market efficiency based on the availability of information. Among the three forms of market efficiency, most of the studies have attended to the weak form of market efficiency which proposes that current stock prices reflect all information contained in the past stock prices. The weak form of market efficiency hypothesis has been tested by Fama for U.S., Dryden (1970) for U.K., Andersen and Bollerslev (1997) for eight European markets, Conrad and Juttner (1973) for Germany, Jennergren and Korsvold (1975) for Norway and Sweden, Lawrence (1986) for Malaysia and Singapore. These studies provided indecisive results. The developed markets, e.g., U.S. as well as some of the European markets were found to be weak form efficient. However, evidence from emerging markets indicated rejection of the weak form market efficiency hypothesis. Therefore question arises whether the returns in such markets is predictable. Apart from the form of efficiency, it is the volatility prevailing in the market which influences the return to a large extent. Volatility, which refers to the degree of unpredictable change over time and might be measured by the standard deviation of a sample, often used to quantify the risk of the instrument of portfolio over that time period. Equity return volatility may be defined as the standard deviation of daily equity returns around the mean value of the equity return and the stock market volatility is the return volatility of the aggregate market portfolio. Engle (1982) in his seminal work introduced the concept of Autoregressive Conditional Heteroscedasticity (ARCH) which became a very powerful tool in the modelling of high frequency financial data in general and stock returns in particular. As compared to conventional time series models, ARCH models allow the conditional variances to change through time as functions of past errors. One significant improvement was introduced by Bollerslev (1986) where the Generalized Autoregressive Conditional Heteroscedasticity (GARCH) process was presented. Further, many more variation

were introduced such as Integrated GARCH (IGARCH) by Engle and Bollerslev (1994) and the exponential GARCH (EGARCH) by Nelson (1991) where re-specification of variance equation was studied.

In financial risk management, Value at Risk (VaR) is widely used as the risk measure and is defined as the maximum potential loss that would be incurred at a given probability p for a financial instrument or portfolio during a given period of time. In general, VaR is calculated either based on Historical Simulation (HS) approach, which imposes virtually no structure on the distribution of returns except stationarity, or using Monte Carlo simulation (MCS) approach which assumes parametric models for variance and subsequently large random numbers are drawn from this specific distribution to calculate the desired risk measure. Filtered Historical Simulation (FHS) approach attempts to combine the best of the model-based with the best of the model-free approaches in a very intuitive fashion.

There are some significant empirical researches on stock return volatility in emerging markets like India in recent time. However, there is hardly any study which estimated VaR following Filtered Historical Simulation approach using GARCH model with suitable mean specification, in the context of Indian capital market. Pattanaik & Chatterjee (2000) used ARCH/GARCH models to model the volatility in Indian financial market. Agarwal and Du (2005) using BSE 200 data have found that the Indian stock market is integrated with the matured markets of the World. Chattopadhyay and Behera (2006) examined whether reforms in Indian stock market have led to integration with the developed stock markets in the world and suggested that Indian stock market is not co-integrated with the developed market as yet although some short-term impact does exist. Moreover, the study also does not find any causality between the Japanese stock market and Indian stock market. More recently, Janak Raj and Sarat Dhal (2008) investigated the financial integration of India's stock market with that of global and major regional markets. They have used six stock price indices i.e. the 200-scrip index of BSE of India to represent domestic market, stock price indices of Singapore and Hong Kong to represent the regional markets and three stock price indices of U.S., U.K. and Japan to represent the global markets. Based on daily as well as weekly data covering end-March 2003 to end-January 2008 they found that Indian market's dependence on global markets, such as U.S. and U.K., was substantially higher than on regional markets such as

Singapore and Hong Kong, while Japanese market had weak influence on Indian market.

The paper examines the financial integration of Indian capital market (BSE-SENSEX and NSE-NIFTY) with other global indicators and its own volatility using daily returns covering the period January 2003 to December 2009. The paper specifies a GARCH framework to model the phenomena of volatility clustering on returns and examines the usefulness of considering lag values of return on (S&P 500, INR-EURO & INR-USD exchange rate, Gold price) as proxies to global financial condition in the specification of the mean equation. The paper also estimate VaR of return in the Indian capital market based on two composite methods i.e. (a) using univariate GARCH model where in the mean equation we have used lag values of return on (S&P 500, INR-EURO & INR-USD exchange rate, Gold price) and following the filtered historical simulation (FHS) approach (b) using ARMA for mean equation, GARCH for volatility and FHS for VaR estimation i.e. ARMA-GARCH-FHS methods; and compare the performance of both the VaR estimates.

The rest of the paper is organised as follows. Section 2 describes the portfolio model using GARCH specifications, section 3 describes estimate of VaR based on HS, MCS and FHS. Section 4 describes the data and focuses on VaR calculation and summarizing the results. Finally, section 5 concludes.

2. The portfolio model

In the financial literature it is well documented that variances of asset returns, in general, changes over time and GARCH models are popular choice to model these changing variances. Let r_t ; $t = 1, \dots, T$, represents the continuously compounded rate of returns of a stock price index (for holding the portfolio for one day) at time t . If p_t is the stock price index then $r_t = \ln(p_t) - \ln(p_{t-1})$, where 'ln' is the natural logarithm. The model can be written as:

$$r_{t+1} = c + \phi_1 r_t + \phi_2 r_{t-1} + \dots + \phi_k r_{t+1-k} + \psi_1 x_{1,t+1} + \psi_2 x_{2,t+1} + \dots + \psi_s x_{s,t+1} + \sigma_{t+1} \eta_{t+1}; \quad t=1,2,\dots,T$$

$$\sigma_{t+1}^2 = \omega + \alpha \text{Resid}_t^2 + \beta \sigma_t^2 \quad (1)$$

where $\text{Resid}_t = (r_t - c - \sum \phi_j r_{t-j} - \sum \psi_j x_{j,t})$; innovation $\{\eta_t\}$ is white noise process, with zero mean and unit variance and $\alpha + \beta < 1$.

3. Value at Risk

Value at Risk is being widely used as measure of market risk of an asset or of a portfolio. The Parametric VaR model imposes a strong theoretical assumption on the underlying properties of data; frequently Normal Distribution is assumed because it is well described, can be defined using only the first two moments and it can be understood easily. Other probability distributions may be used, but at a higher computational cost. However, empirical evidence indicates that asset price changes, in particular the daily price changes, most of the time does not follow Normal Distribution. In the presence of excess kurtosis, failure rate increases when the VaR is estimated by the Gaussian distribution. The $100\alpha\%$ one day ahead VaR ($\lambda_{\alpha,t}$) defined as $P[r_t \leq \lambda_{\alpha,t} | r_{t-1}] = \alpha$. In general, VaR techniques are based on non-parametric or mixture of parametric and non-parametric statistical methods. The family of Historical Simulation (HS) models is a non-parametric approach. The Filtered Historical Simulation (FHS) as developed by Barone-Adesi et al (1998) and Barone-Adesi et al (1999, 2000) is mixture of parametric and non-parametric approach.

3.1. Historical Simulation

Apart from stationarity of the returns, Historical Simulation (HS) does not require any statistical assumption in particular to the volatility. In Historical Simulation method we consider the availability of a past sequence of daily portfolio returns for m days; r_t $t=1,2,\dots,m$. The HS technique simply assumes that the distribution of tomorrow's portfolio returns, r_{t+1} , is well approximated by the empirical distribution of the past m observations—that is, $\{r_{t+1-\tau}\}_{\tau=1..m}$. In other words, the distribution of r_{t+1} is captured by the histogram of $\{r_{t+1-\tau}\}_{\tau=1..m}$. Thus, we simply sort the returns in $\{r_{t+1-\tau}\}_{\tau=1..m}$ in ascending order and choose the VaR_{t+1}^p to be the number such that only $100p\%$ of the observations are smaller than the VaR_{t+1}^p .

3.2 Monte Carlo Simulation (MCS)

MCS can be explained better through an example. Let us consider GARCH(1,1) model as defined in equation (1) i.e.

$$r_{t+1} = c + \phi_1 r_t + \phi_2 r_{t-1} + \dots + \phi_k r_{t+1-k} + \psi_1 x_{1,t+1} + \psi_2 x_{2,t+1} + \dots + \psi_s x_{s,t+1} + \sigma_{t+1} \eta_{t+1}; \quad t=1,2,\dots,T$$
$$\sigma_{t+1}^2 = \omega + \alpha \text{Resid}_t^2 + \beta \sigma_t^2$$

where $\text{Resid}_t = (r_t - c - \sum \phi_j r_{t-j} - \sum \psi_j x_{j,t})$; innovation $\{\eta_t\}$ is white noise process, with zero mean and unit variance and $\alpha + \beta < 1$. Although, in the case of daily asset returns,

generally, η_t does not follow Normal Distribution but since using other probability distributions is computational very costly, let us assume η_t follows Normal Distribution $N(0,1)$.

Based on the above specified GARCH model, at the end of day 't' we can calculate the variance of day 't+1' i.e. σ_{t+1}^2 .

Let $\{\eta_{i,1}^{\wedge}; i=1,2,\dots,L\}$ be a set of large number of random numbers drawn from the standard Normal Distribution $N(0,1)$. From these random numbers $\{\eta_{i,1}; i=1,2,\dots,L\}$ we can calculate a set of hypothetical returns for day 't+1' as

$$r_{i,t+1}^{\wedge} = c + \sum \phi_i r_{t+1-i} + \sum \psi_j x_{t+1-j} + \sigma_{t+1}^{\wedge} \eta_{i,1}^{\wedge}; i=1,2,\dots,L$$

$$\text{Resid}_{i,t+1} = (r_{i,t+1}^{\wedge} - c - \sum \phi_i r_{t+1-i} - \sum \psi_j x_{t+1-j})$$

Given these hypothetical returns ($r_{i,t+1}^{\wedge}$) for day 't+1', we can compute the hypothetical variances for the 't+2' day as

$$\sigma_{t+2}^2 = \omega + \alpha \text{Resid}_{t+1}^2 + \beta \sigma_{t+1}^2$$

Similarly, to estimate the hypothetical return ($r_{i,t+2}^{\wedge}$) on day $t + 2$, draw again large number of pseudo random numbers from the $N(0, 1)$ distribution i.e. $\{\eta_{i,2}; i=1,2,\dots,L\}$

$$r_{i,t+2}^{\wedge} = c + \sum \phi_i r_{t+2-i} + \sum \psi_j x_{t+2-j} + \sigma_{t+2}^{\wedge} \eta_{i,2}^{\wedge}; i=1,2,\dots,L$$

$$\text{Resid}_{i,t+2} = (r_{i,t+2}^{\wedge} - c - \sum \phi_i r_{t+2-i} - \sum \psi_j x_{t+2-j})$$

and variance is now updated by

$$\sigma_{t+3}^2 = \omega + \alpha \text{Resid}_{t+2}^2 + \beta \sigma_{t+2}^2$$

Similarly we can get the hypothetical return of 't+k' day

$$r_{i,t+k}^{\wedge} = c + \sum \phi_i r_{i,t+k-1} + \sum \psi_j x_{j,t+k-1} + \sigma_{t+k-1}^{\wedge} \eta_{i,k}^{\wedge}; i=1,2,\dots,L$$

Therefore, hypothetical K -day return can be written as

$$r_{i,t+1:t+k}^{\wedge} = \sum_k r_{i,t+k}^{\wedge}; i=1,2,\dots,L$$

If we collect these L hypothetical K -day returns in a set $\{r_{i,t+1:t+k}^{\wedge}; i=1,2,\dots,L\}$, then the K -day VaR can be calculated as the 100p percentile i.e.

$$\text{VaR}_{t+1:t+k}^p = - \text{Percentile}\{\{r_{i,t+1:t+k}^{\wedge}; i=1,2,\dots,L\}, 100p\}$$

3.3 Filtered Historical Simulation (FHS)

As we have discussed that non-parametric approach such as Historical Simulation (HS) does not assume any statistical distribution of returns, whereas parametric

approach such as the Monte Carlo simulation (MCS) takes the opposite view and assumes parametric models for variance, correlation (if a disaggregate model is estimated), and the distribution of standardized returns. Random numbers are then drawn from this distribution to calculate the VaR. Both of these extremes in the model-free/model-based spectrum have pros and cons. MCS is good if the assumed distribution is fairly accurate in description of reality. HS is sensible as the observed data may capture features of the returns distribution that are not captured by any standard parametric model. The FHS approach on the other hand attempts to combine the best of the MCS with the best of the HS.

Let's assume we have estimated a GARCH-type model of our portfolio variance given in equation (1). Although we are comfortable with our variance model (σ), we are not comfortable making a specific distributional assumption about the (η), such as a Normal or a t distribution. Instead of that, we might like the past returns data (r_t) to determine the distribution directly without making further assumptions.

Given a sequence of past returns and estimated GARCH volatility, $\{r_{t+1-\tau}, \sigma_{t+1-\tau}^{\wedge}; \tau = 1, 2, \dots, m\}$ and , calculated past standardized returns are given by

$$\eta_{t+1-\tau}^{\wedge} = (r_{t+1-\tau} - E(r_{t+1-\tau})) / \sigma_{t+1-\tau}^{\wedge}; \tau = 1, 2, \dots, m$$

Instead of drawing random η^{\wedge} 's from a specific probability distribution as it is done in MCS, in FHS method samples are drawn with replacement from $\{\eta_{t+1-\tau}^{\wedge}; \tau = 1, 2, \dots, m\}$.

Thereafter, similarly as in section 3.2, we can get the hypothetical return of 't+k' day

$$r_{i,t+1:t+k}^{\wedge} = c + \sum \phi_i r_{i,t+k-1} + \sum \psi_j x_{j,t+k-1} + \sigma_{t+k-1}^{\wedge} \eta_{i,k}^{\wedge}; i=1, 2, \dots, L.$$

Therefore, hypothetical K -day return can be written as

$$r_{i,t+1:t+k}^{\wedge} = \sum_k r_{i,t+k}^{\wedge}; i=1, 2, \dots, L$$

The K -day VaR can be calculated based on L estimated k -day returns $\{r_{i,t+1:t+k}^{\wedge}\}$ as the 100p percentile i.e.

$$VaR_{t+1:t+k}^p = - \text{Percentile}\{\{r_{i,t+1:t+k}^{\wedge}; i=1, 2, \dots, L\}, 100p\}$$

4. Empirical results

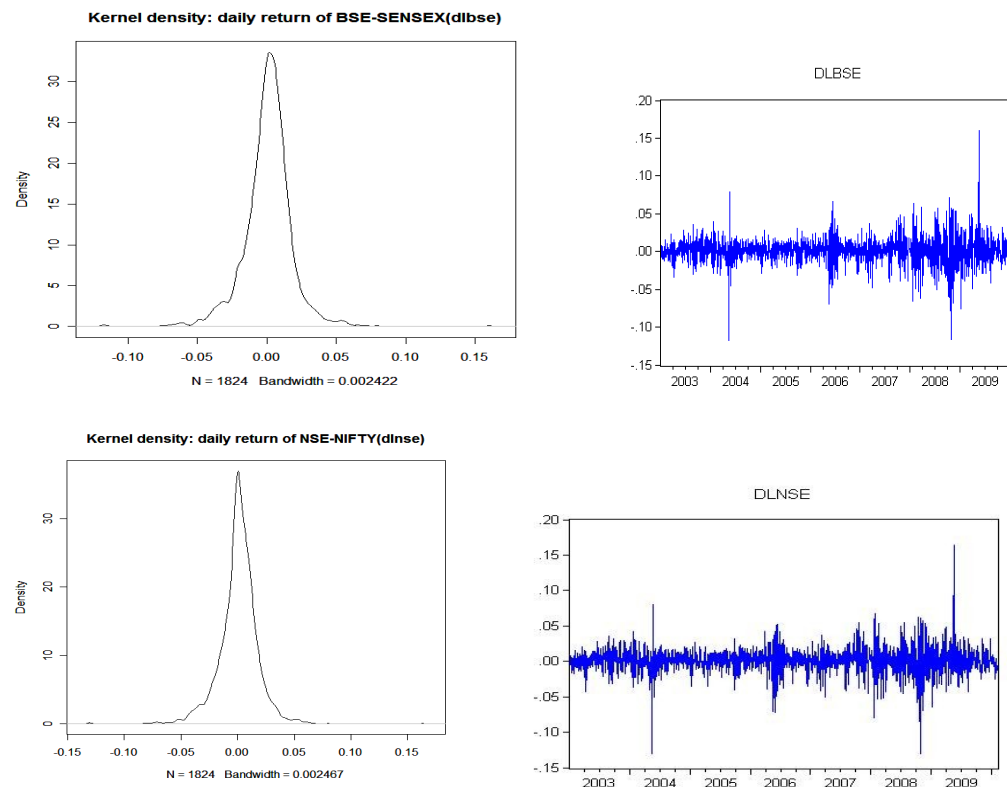
In this study, we have used daily data of two stock price indices viz. BSE-SENSEX (BSE) and NSE-NIFTY (NSE) covering the period from January 2003 to December 2009. We have estimated 1-day VaR for the daily returns of two

price indices using univariate GARCH model with proper mean specification and following the FHS approach for VaR estimation. We have also estimated VaR of return using ARMA-GARCH-FHS model and compare the performance of both the VaR estimate. We have used daily S&P500 stock price (SP), daily exchange rate of INR-USD (usd), INR-EURO (euro) and also the gold prices in INR/ounce (gold) for the same period as explanatory variable of the mean equation of the stock prices return. Unit root test (ADF,PP test) suggest that level series of all the six data series are non-stationary, however, continuous daily return i.e. log differences of the series (dlbse, dlNSE, dlSP, dlUSD, dlEURO and dlGOLD) are stationary.

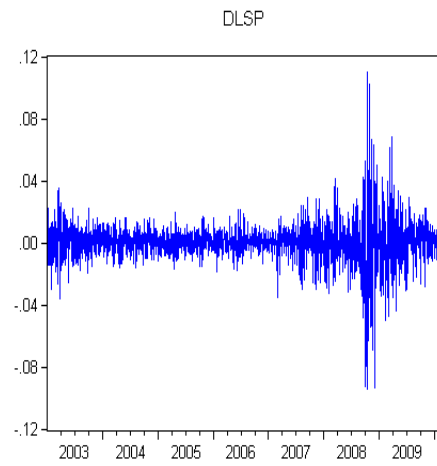
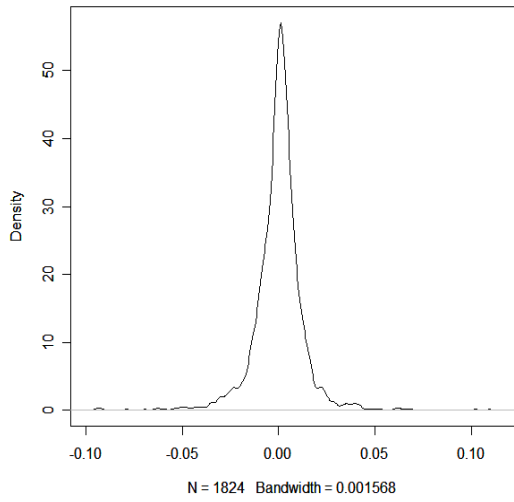
4.1. Stylised facts

Continuous daily return (log difference) and kernel density of returns on BSE-SENSEX, NSE-NIFTY, S&P500, INR-USD exchange rate, INR-EURO exchange rate and Gold prices for the reference period are given in chart 1 and descriptive statistics are given in table 1. There is a clear presence of fat tails in the return distribution of all the six data series.

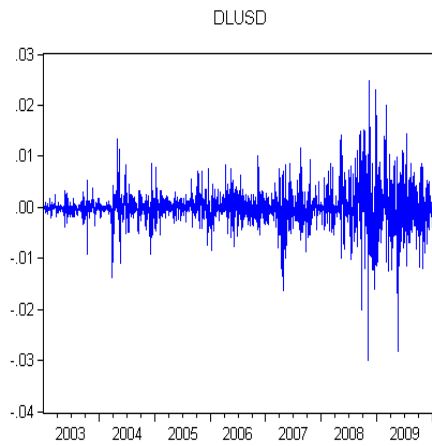
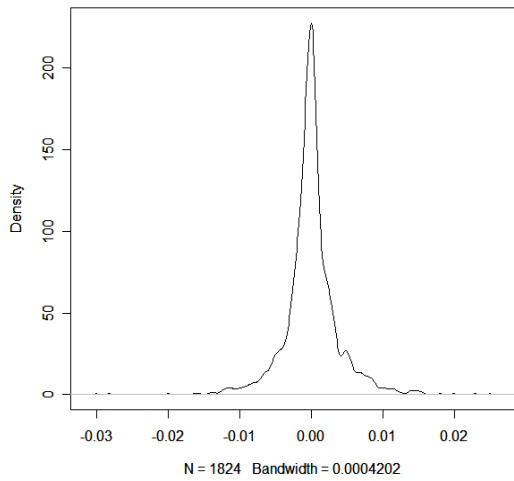
Chart 1: Plot of daily returns and kernel density of daily returns of six series



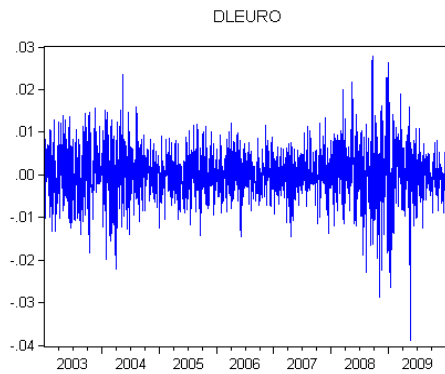
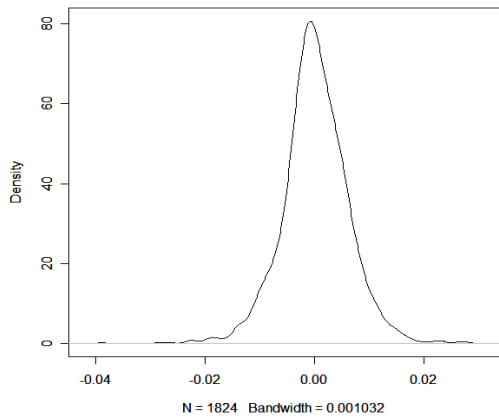
Kernel density: daily return of S&P 500 stockprices(dlsp)



Kernel density: daily return of INR-USD exchange rate(dlUSD)



Kernel density: daily return of INR-EURO exchange rate(dleuro)



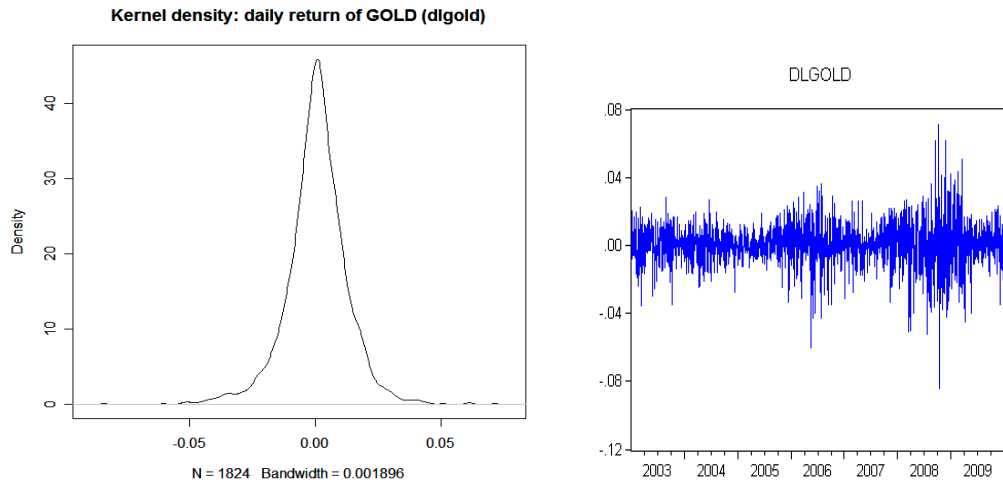


Table 1: Descriptive statistics

	DLNSE	DLBSE	DLSP	DLUSD	DLEURO	DLGOLD
Mean	0.000804	0.000849	0.000105	-2.02E-05	0.000134	0.000617
Median	0.001078	0.001614	0.000799	0	0	0.000865
Maximum	0.163343	0.1599	0.109572	0.024903	0.0279	0.071278
Minimum	-0.13054	-0.11809	-0.0947	-0.03007	-0.03889	-0.08396
Std. Dev.	0.017498	0.017166	0.013291	0.003855	0.006085	0.012485
Skewness	-0.31933	-0.11242	-0.23195	-0.02245	-0.14065	-0.30721
Kurtosis	12.07311	11.0435	15.13967	10.82578	5.58351	6.944578
Jarque-Bera	6370.147	4985.629	11364.19	4715.852	520.0318	1211.227
Sum	1.485895	1.569306	0.193701	-0.03735	0.24755	1.126134
Sum Sq. Dev.	0.56552	0.544288	0.326289	0.027452	0.068397	0.284156
Observations	1848	1848	1848	1848	1848	1824

4.2 Modelling Volatility

Equation (2) and (3) presents the estimated portfolio model where lag values of (dlbse, dlsp, dlusd, dleuro and dlgold) are used in the mean equation of the GARCH(1,1) model of BSE and NSE respectively.

Eq (2):

$$\begin{aligned}
 \mathbf{D}(\mathbf{LOG}(\mathbf{BSE})) = & 0.00152 + 0.32558 \cdot \mathbf{D}(\mathbf{LOG}(\mathbf{SP500}(-1))) + \\
 & \quad (0.00027) \quad \quad \quad (0.02662) \\
 & 0.16716 \cdot \mathbf{D}(\mathbf{LOG}(\mathbf{SP500}(-2))) + 0.13393 \cdot \mathbf{D}(\mathbf{LOG}(\mathbf{SP500}(-3))) + \\
 & \quad (0.026299) \quad \quad \quad (0.029617) \\
 & 0.10005 \cdot \mathbf{D}(\mathbf{LOG}(\mathbf{SP500}(-4))) - 0.06044 \cdot \mathbf{D}(\mathbf{LOG}(\mathbf{BSE}(-2))) - \\
 & \quad (0.027628) \quad \quad \quad (0.025798) \\
 & 0.04891 \cdot \mathbf{D}(\mathbf{LOG}(\mathbf{BSE}(-3))) + 0.05722 \cdot \mathbf{D}(\mathbf{LOG}(\mathbf{GOLD}(-2))) + \\
 & \quad (0.020935) \quad \quad \quad (0.023705) \\
 & 0.15824 \cdot \mathbf{D}(\mathbf{LOG}(\mathbf{EURO}(-3))) - 0.24620 \cdot \mathbf{D}(\mathbf{LOG}(\mathbf{USD}(-3))) + \\
 & \quad (0.053619) \quad \quad \quad (0.088501) \\
 & 0.15767 \cdot \mathbf{D}(\mathbf{LOG}(\mathbf{USD}(-4))) \\
 & \quad (0.084305)
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{GARCH} = & 5.049\text{e-}06 + 0.1527939 \cdot \mathbf{RESID}(-1)^2 + 0.83803454 \cdot \mathbf{GARCH}(-1) \\
 & \quad (5.05\text{E-}06) \quad \quad (0.152794) \quad \quad \quad (0.838035)
 \end{aligned}$$

Eq (3):

$$\begin{aligned}
 D(\text{LOG}(\text{NSE})) &= 0.00134 + 0.32468 * D(\text{LOG}(\text{SP500}(-1))) + \\
 &\quad (0.000292) \quad (0.027540) \\
 &+ 0.16821 * D(\text{LOG}(\text{SP500}(-2))) + 0.10268 * D(\text{LOG}(\text{SP500}(-3))) - \\
 &\quad (0.027508) \quad (0.030620) \\
 &+ 0.03877 * D(\text{LOG}(\text{NSE}(-2))) + 0.06159 * D(\text{LOG}(\text{GOLD}(-2))) + \\
 &\quad (0.026039) \quad (0.025277) \\
 &+ 0.17716 * D(\text{LOG}(\text{EURO}(-3))) - 0.34938 * D(\text{LOG}(\text{DOLLAR}(-3))) + \\
 &\quad (0.051925) \quad (0.088119) \\
 &+ 0.19060 * D(\text{LOG}(\text{DOLLAR}(-4))) \\
 &\quad (0.087568) \\
 \text{GARCH} &= 5.52699\text{e-}06 + 0.13072 * \text{RESID}(-1)^2 + 0.85739 * \text{GARCH}(-1) \\
 &\quad (5.53\text{E-}06) \quad (0.130725) \quad (0.857389)
 \end{aligned}$$

Equation (4) and (5) presents the estimated portfolio model using ARMA-GARCH model of BSE and NSE respectively.

Eq (4):

$$\begin{aligned}
 D(\text{LOG}(\text{BSE})) &= 0.00161 + [\text{AR}(1)=0.52534, \text{AR}(2)=-0.87026, \\
 &\quad (0.000314) \quad (0.062328) \quad (0.061490) \\
 &\text{MA}(2)=0.79823, \text{MA}(3)=0.12583, \text{MA}(1)=-0.42263] \\
 &\quad (0.066763) \quad (0.026230) \quad (0.066514) \\
 \text{GARCH} &= 6.28919\text{e-}06 + 0.15594 * \text{RESID}(-1)^2 + 0.83073 * \text{GARCH}(-1) \\
 &\quad (6.29\text{E-}06) \quad (0.155941) \quad (0.830730)
 \end{aligned}$$

Eq (5):

$$\begin{aligned}
 D(\text{LOG}(\text{NSE})) &= 0.00160 + [\text{AR}(2)=-0.45572, \text{AR}(4)=-0.6135 \\
 &\quad (0.000317) \quad (0.084090) \quad (0.096555) \\
 &\text{AR}(1)=0.49844, \text{MA}(2)=0.42253, \text{MA}(4)=0.67288, \text{MA}(1)=-0.43386] \\
 &\quad (0.134215) \quad (0.079148) \quad (0.092978) \quad (0.127459) \\
 \text{GARCH} &= 7.61169\text{e-}06 + 0.13774 * \text{RESID}(-1)^2 + 0.84330 * \text{GARCH}(-1) \\
 &\quad (1.12\text{E-}06) \quad (0.011880) \quad (0.012652)
 \end{aligned}$$

*Values given in () are the standard error

4.3 Value at Risk: results

We have estimated 5% 1-day-VaR for both BSE-SENSEX and NSE-NIFTY daily retrn using univariate GARCH model with proper mean specification as estimated in section 4.2 and following the FHS approach for VaR estimation (Model A). We have also estimated 5% VaR for both BSE-SENSEX and NSE-NIFTY daily return using ARMA-GARCH-FHS model (Model B). To estimate the model parameter we have used the data from 2nd January 2003 to 30th October 2009 and forecasted dynamically 1-day VaR for the period 2nd November 2009 to 24th December 2009 i.e. for 39 days. Actual returns and forecasted VaR based on both Model A and Model B for BSE-SENSEX and NSE-NIFTY are given in chart 2 and chart 3, respectively. Out of 39 forecasts of VaR for BSE and NSE, only in one occasion actual return was less than the VaR estimate (failure rate 1/39) for both model A and model B. However, dispersion of VaR from actual returns is not the same.

Let the dispersion of VaR at 5% significant level based on model A (${}^A\text{VaR}_t^{.05}$) from the actual return (r_t) be $D^A = \sum (r_t - {}^A\text{VaR}_t^{.05})^2$ and $D^B = \sum (r_t - {}^B\text{VaR}_t^{.05})^2$ for mode B. It is observed that ($D^A_{\text{BSE}} = 0.02625$, $D^B_{\text{BSE}} = 0.03022$), ($D^A_{\text{NSE}} = 0.02673$, $D^B_{\text{NSE}} = 0.03050$) Since $D^A_{\text{BSE}} < D^B_{\text{BSE}}$; $D^A_{\text{NSE}} < D^B_{\text{NSE}}$, we conclude that for both BSE-SENSEX and NSE-NIFTY price indices, model A performs better in estimating the VaR.

Chart 2: Daily return on BSE and corresponding VaR based on Model A and Model B

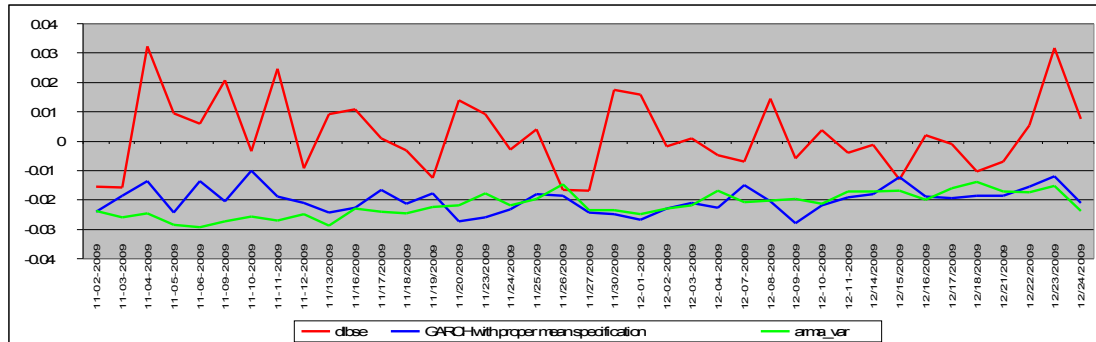
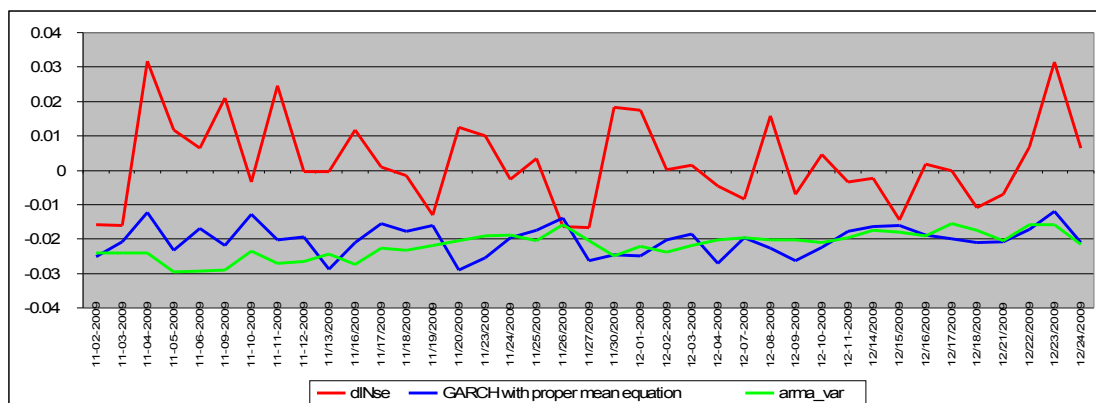


Chart 3: Daily return on NSE and corresponding VaR based on Model A and Model B



5. Conclusion

The paper estimate 1-day VaR taking into consideration the financial integration of Indian capital market (BSE-SENSEX and NSE-NIFTY) with other global indicators and its own volatility using daily return covering the period January 2003 to December 2009. The paper specifies a GARCH framework to model the phenomena of volatility clustering on returns and examines the usefulness of considering lag values of return on (S&P 500, INR-EURO & INR-USD exchange rate, Gold price) as proxies to global financial condition in the specification of the mean equation. The paper estimate the VaR of return in the Indian capital market based on two composite methods i.e. (a) using univariate GARCH model where in the mean equation we have used lag values of return on (S&P 500, INR-EURO & INR-USD exchange rate, Gold price) and following the FHS approach (b) using ARMA-GARCH-FHS; and compare the performance of both the VaR estimate. It is found that VaR of return in the Indian capital market estimated based on method (a) i.e. GARCH with proper mean specification performs better than method; and (b) using ARMA for mean equation, GARCH for volatility and FHS for VaR estimation i.e. ARMA-GARCH-FHS; and compared the performance of the VaR estimate from both the methods. Empirically, it is found that global financial situation (lag values of return on S&P 500, INR-EURO & INR-USD exchange rate, Gold price used as proxies to global financial condition) has significant impact on Indian capital market. Also estimated VaR of return in the Indian capital market estimated based on GARCH method with suitable mean specification outperforms the ARMA-GARCH method.

References

- Barone-Adesi G, K Giannopoulos and L Vosper, 1999, "VaR Without Correlations for Non-Linear Portfolios", *Journal of Futures Markets*, 19, August, 583-602.
- Barone-Adesi G, K Giannopoulos and L Vosper, 2000, "Filtered Historical Simulation. Backtest Analysis"
- Barone-Adesi G, Rober F. Engle, Lorian Manchini, 2008, " A GARCH option pricing model with Historical Filtered Simulation"
- Bollerslev T. 1986. Generalized Autoregressive Conditional Heteroscedasticity *Journal of Econometrics*. 31(3)
- Bollerslev T. 1987. A Conditionally Heteroskedastic Time Series Model for Speculative Prices and Rates of Return *Review of Economics and Statistics* 69(4)
- Conrad K and D Juttner. 1973. Recent behavior of Stock Market prices in Germany and the Random Walk Hypothesis. *Kyklos*. 26(5)
- Dryden M. 1970. A Statistical study of UK share prices. *Scottish Journal of Political Economy*. 56(2)
- Engle R F. 1982. Autoregressive Conditional Heteroskedasticity with Estimates of the variance of United Kingdom Inflation. *Econometrica*. 50(4)
- Engle R F and K Kroner. 1995. Multivariate Simultaneous Generalized ARCH *Econometric Theory*. 11(1)
- Engle R F, V Ng and M Rothschild. 1990. Asset Pricing with a Factor ARCH Covariance Structure: Empirical Estimates for Treasury Bills. *Journal of Econometrics*. 45 (2)
- Fama E. 1970. Efficient Capital Markets: A Review of Theory and Empirical Work. *Journal of Finance*. 45 (2)
- Garman M B and M J Klass. 1980. On Estimation of Security Price Volatilities from Historical Data. *Journal of Business*. 53(1)
- Hotta L K, Lucas E C, Palaro H P, " Estimation of VaR using Copula and Extreme Value Theory" , *Multinational Financial Journal – Vol. 12, Nbr. 3/4 September 2008*
- Janak Raj, Sarat Dhal, (2008) " Integration of India's stock market with global and major regional markets", *BIS paper No. 42*.
- Johansen, S (1988): "Statistical analysis of cointegrating vectors", *Journal of Economic Dynamics and Control*, vol 12, pp 231–54.
- Jeroen V K, Rombouts and Marno Verbeek, (2009), "Evaluating Portfolio Value-at-Risk Using Semi-Parametric GARCH Models", *ERIM Report Series in Management*.
- Kaushik Bhattacharya, Nityananada Sarkar, Debabrata Mukhopadhyay,(2003), "Stability of the Day of the Week Effect in Return and Volatility at the Indian Capital Market: A GARCH Approach with Proper Mean Specification", *Applied Financial Economics*, 13.
- Nath G C and S Verma (2003): "Study of Common Stochastic Trend and Cointegration in the Emerging Markets - A Case Study of India, Singapore and Taiwan", *NSE Working Paper No. 25*.
- Peter F. Christoffersen, (2003), "Elements of Financial Risk Management". Academic Press.