Financial inclusion, productivity shocks and consumption volatility in emerging economies

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Abstract

How does access to finance impact consumption volatility? Theory and evidence from advanced economies suggests that greater household access to finance smooths consumption. Evidence from emerging markets, where consumption is usually more volatile than income, indicates that financial reform further increases the volatility of consumption relative to output. We address this puzzle in the framework of an emerging economy model in which households face shocks to trend growth rate, and a fraction of them are financially constrained, with no access to financial services. Unconstrained households can respond to shocks to trend growth by raising current consumption more than the rise in current income. Financial reform increases the share of such households, leading to greater relative consumption volatility. Calibration of the model for pre and post financial reform in India provides support for the model’s key predictions.

JEL Classification: E10, E32
Keywords: Macroeconomics, Real Business Cycles, Emerging Market business cycle stylized facts, financial development.

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1 Introduction

Emerging economies have been seen to witness an increase in consumption volatility relative to output volatility after financial development. This behaviour appears puzzling since traditional models and evidence from advanced economies suggests that consumption should become smoother after financial constraints are reduced. This puzzle can be explained in a model featuring financial constraints and shocks to trend growth of productivity. The model predicts that relative consumption volatility rises when more consumers can access financial services.

The presence of financial constraints, such as credit constraints or lack of access to financial services in an economy explain the excess volatility of consumption and its sensitivity to anticipated income fluctuations. A model featuring financially constrained consumers predicts that consumption cannot be smoothed fully. But in such a model, the volatility of consumption can be at least as high as income volatility, or at most one. Further, if constraints are eased the model predicts a reduction in relative consumption volatility.

Another feature of emerging economy models is the presence of shocks to trend growth of productivity. Large shocks to the permanent component of income originated from frequent policy regime shifts in emerging economies, relative to transitory income shocks, explain larger fluctuations in consumption relative to output fluctuations (Aguiar and Gopinath, 2007). Unlike developed countries characterised by large transitory movements in income around the trend, shocks to trend growth are the primary source of fluctuations in emerging economies. When households anticipate a higher growth rate of income which eventually leads to a rise in future income, they respond to this permanent income shock by increasing current consumption more than the rise in current income via borrowing against the future income or reducing current savings. As a result, consumption fluctuates more than income in emerging economies. This feature results in the relative volatility of consumption in emerging economies becoming greater than one.

A common feature of reform in emerging economies is financial sector reform. The increase in the access of households to finance resulting from reform allows households to smooth consumption over their lifetimes. But at the same time, emerging economies witness large shocks to the permanent component of income, relative to transitory income shocks. The combination of the response of households to permanent income shocks and the easing of financial constraints can yield an increase in the relative volatility of consumption.
The goal of this paper is to understand the joint impact of easing of financial constraints and permanent income shock on consumption volatility. We present a model in which some households do not have access to finance. They can neither save nor borrow. These financially constrained households cannot smooth consumption over their lifetimes. The rest of the households in the economy are unconstrained and respond to a perceived income shock by smoothing consumption. Shocks to income that are perceived to be permanent lead to an increase in current period consumption higher than the increase in current period income. Only unconstrained households can increase consumption by more than the increase in income, either by borrowing against future income or reducing current savings. Constrained households can only increase consumption by the amount income has increased. Financial sector reform allows more households to access financial services. Now more households become unconstrained and can respond to the income shock that they perceive to be permanent. The key prediction of this model is that financial development in an emerging economy leads to an increase in relative consumption volatility.

This prediction can be tested. We calibrate the model to Indian data for pre and post reform years, where we keep all other parameters constant and only change the share of financially constrained consumers. We find support for our model’s key prediction.

Our paper makes a contribution towards understanding the joint impact of financial development and permanent income shock on consumption volatility. It contributes to a growing literature that studies the effects of financial frictions on volatility. Earlier work mainly analyses the effect of domestic financial system development on output and consumption volatility through its effect on firms (Aghion et al., 2010). Some papers focus on the impact of financial globalisation on volatility (Buch et al., 2005; Leblebicioglu, 2009). The effect of domestic financial system development on output and consumption volatility is explored in a limited strand of literature. Iyigun and Owen (2004) propose a theory of income inequality in rich and poor countries as the cause of consumption volatility whose mechanics partly resemble those of our model, once appropriately re-interpreted.

Our model takes into account the broadly acknowledged fact that in emerging economies all consumers do not have access to finance (Honohan, 2006). Financially constrained households are modelled as in Hayashi (1982); Campbell and Mankiw (1991). The framework includes shocks to trend growth as in Aguiar and Gopinath (2007).

The rest of the paper is organised as follows. Section 2 presents evidence
on relative consumption volatility and financial development in emerging economies. Section 3 discuss the role of the relative magnitude of permanent and transitory income shocks for consumption volatility in developed vis a vis emerging economies. Section 4 presents the model and its predictions. Section 5 contains the calibration exercise and results. Section 6 presents the implications in small open economy set up. Section 7 concludes.

2 Consumption volatility and financial development

Table 1 Relative consumption volatility: Selected emerging economies

<table>
<thead>
<tr>
<th>Region &amp; reform date</th>
<th>Relative consumption volatility</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Pre-reform</td>
</tr>
<tr>
<td>Latin America: 1990</td>
<td></td>
</tr>
<tr>
<td>Chile</td>
<td>1.10</td>
</tr>
<tr>
<td>Colombia</td>
<td>0.97</td>
</tr>
<tr>
<td>Mexico</td>
<td>0.94</td>
</tr>
<tr>
<td>Peru</td>
<td>1.09</td>
</tr>
<tr>
<td>East Asia: 1996</td>
<td></td>
</tr>
<tr>
<td>Indonesia</td>
<td>2.45</td>
</tr>
<tr>
<td>Malaysia</td>
<td>1.36</td>
</tr>
<tr>
<td>Philippines</td>
<td>0.73</td>
</tr>
<tr>
<td>Korea</td>
<td>0.93</td>
</tr>
<tr>
<td>Taiwan</td>
<td>1.84</td>
</tr>
<tr>
<td>Thailand</td>
<td>0.88</td>
</tr>
<tr>
<td>East Europe: 1990</td>
<td></td>
</tr>
<tr>
<td>Turkey</td>
<td>1.07</td>
</tr>
<tr>
<td>Poland</td>
<td>0.92</td>
</tr>
<tr>
<td>Hungary</td>
<td>1.01</td>
</tr>
<tr>
<td>South Asia</td>
<td></td>
</tr>
<tr>
<td>India: 1992</td>
<td>0.83</td>
</tr>
<tr>
<td>Africa</td>
<td></td>
</tr>
<tr>
<td>South Africa: 1994</td>
<td>1.42</td>
</tr>
<tr>
<td>Mean</td>
<td>1.15</td>
</tr>
<tr>
<td>Std. dev.</td>
<td>0.44</td>
</tr>
</tbody>
</table>

Source: Datastream, Author’s calculations

Recent empirical evidence on emerging economy business cycles shows an increase in the volatility of consumption relative to that of output after financial sector reform in Asia, Turkey and in India (Kim et al., 2003; Alp et al., 2012; Ghate et al., 2013).
We measure relative volatility of consumption in the pre and post financial sector reform period for some developing countries. The choice of the date on which reform took place is based on Kim et al. (2003); Singh et al. (2005); Rodrik (2008); Alp et al. (2012); Aslund (2012). The analysis is based on annual data for a set of emerging economies. The span of the analysis varies across countries given the availability of the data. Table A.1 in Appendix A lists period of analysis for each country. The reform date for each region, and the sources of the documentations indicating the reform dates are also reported in this table.

Table 1 shows the reform date and the volatility of consumption relative to that of output in the pre and post reform period. It shows that many emerging economies exhibit similar behaviour in that relative consumption volatility increases after reform.

Financial development has been a major component of reform. Table 2 shows the density of commercial bank branches and depositors with commercial banks in the beginning and in the end of the last decade in emerging economies. It indicates an increase in access of households to finance.

Figure 1 shows the trend in a commonly used indicator of financial development, namely, total bank deposits to GDP for a set of emerging economies. It indicates an increase in access of households to finance.

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Table 2 Access to finance

<table>
<thead>
<tr>
<th>Country</th>
<th>Commercial bank branches per 100,000 adults</th>
<th>Depositors with commercial banks per 1,000 adults</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chile</td>
<td>13</td>
<td>18</td>
</tr>
<tr>
<td>Colombia</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mexico</td>
<td>11</td>
<td>15</td>
</tr>
<tr>
<td>Peru</td>
<td>5</td>
<td>8</td>
</tr>
<tr>
<td>Indonesia</td>
<td>13</td>
<td>8</td>
</tr>
<tr>
<td>Malaysia</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>Korea</td>
<td>17</td>
<td>19</td>
</tr>
<tr>
<td>Taiwan</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Thailand</td>
<td>8</td>
<td>11</td>
</tr>
<tr>
<td>Turkey</td>
<td>13</td>
<td>..</td>
</tr>
<tr>
<td>Poland</td>
<td>37</td>
<td>46</td>
</tr>
<tr>
<td>Hungary</td>
<td>14</td>
<td>17</td>
</tr>
<tr>
<td>India</td>
<td>10</td>
<td>11</td>
</tr>
<tr>
<td>South Africa</td>
<td>5</td>
<td>10</td>
</tr>
</tbody>
</table>

Source: Financial Inclusion, World Development Indicators

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1The set of emerging economies consists of Chile, Columbia, Mexico, Peru, Indonesia,
Figure 1 Financial development

This figure shows the average deposits to GDP ratio of a set of emerging economies and a few individual countries in the set.

Source: International Financial Statistics, IMF.

shows a rise in the average bank deposits to GDP ratio for the set of emerging economies. The rising trend in the ratio is also visible for individual countries (Figure 1).

The above evidence suggests that the relative volatility of consumption rises after financial sector reform. This appears puzzling and cannot be explained by the existing literature. It supports the evidence in Kim et al. (2003); Alp et al. (2012); Ghate et al. (2013) who allude to the increase in relative consumption volatility after financial sector reform.

3 Consumption volatility and permanent versus transitory income shocks

Empirical literature on business cycle stylised facts document business cycle properties in developed economies (Kydland and Prescott, 1990; Backus and Kehoe, 1992; Stock and Watson, 1999; King and Rebelo, 1999) and developing countries (Agenor et al., 2000; Rand and Tarp, 2002; Male, 2010). One of the key business cycle features that distinguishes emerging economies from advanced countries is the greater fluctuations in consumption relative to income fluctuations. Aguiar and Gopinath (2007) relate this difference in Malaysia, Philippines, Korea, Taiwan, Thailand, Turkey, Poland, Hungary, India, South Africa.
consumption behaviour in the two sets of countries to the relative magnitude of permanent and transitory shocks to income.

The authors estimate a standard small open economy real business cycle model for Mexico as a representative of the emerging economies, and Canada, representing advanced countries. They find that large shocks to the growth rate of the permanent component of productivity are the primary sources of fluctuations in emerging economies. In contrast, advanced economies are characterised by fluctuations around a stable trend, caused by large shocks to transitory component of productivity. The differences in technology shock processes cause households to respond differently to income shocks in developed and emerging economies. When households anticipate a higher growth rate of income which eventually leads to a rise in future income, they respond to this permanent income shock by increasing current consumption more than the rise in current income via borrowing against the future income or reducing current savings. As a result, consumption fluctuates more than income in emerging economies. This feature results in the relative volatility of consumption in emerging economies to be greater than one.

Table 3 document a positive correlation between the magnitude of shocks to trend growth and relative consumption volatility. The second to fifth columns of the table show that technological shock processes for Mexico and Canada, along with output and consumption volatilities estimated from the model in Aguiar and Gopinath (2007). The second and fourth columns also document the empirical volatilities in output and consumption for these two countries. The table shows that Mexico, with consumption volatility relative to output volatility greater than one, is characterised by a larger shock to the growth rate of permanent component of technology $\sigma_g$, compared to the transitory shock $\sigma_a$. In contrast, Canada with a relative consumption volatility less than one, is characterised by larger transitory shocks compared to fluctuation in the permanent component of productivity.

Similarly, Naoussi and Tripier (2013) estimate a real business cycle model with transitory and trend shocks on productivity for 82 countries including developed, emerging and Sub-Saharan African (SSA) countries. They find that magnitude of trend shocks are positively correlated with relative consumption volatilities. Columns 6 to 11 in Table 3 depict their findings. Relative consumption volatilities and shock to trend growth rate are found to be highest for SSA countries, followed by emerging and developed economies.
Table 3 Comparing cross country technology shock processes

<table>
<thead>
<tr>
<th></th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Data</td>
<td>Model</td>
<td>Data</td>
<td>Model</td>
<td>Data</td>
</tr>
<tr>
<td>$\sigma_y$</td>
<td>2.40</td>
<td>2.13–2.40</td>
<td>1.55</td>
<td>1.24–1.55</td>
<td>2.25</td>
</tr>
<tr>
<td>$\sigma_c$</td>
<td>3.02</td>
<td>3.02–3.27</td>
<td>1.15</td>
<td>0.94–1.41</td>
<td>2.33</td>
</tr>
<tr>
<td>$\sigma_c/\sigma_y$</td>
<td>1.26</td>
<td>1.10–1.33</td>
<td>0.74</td>
<td>0.74–0.91</td>
<td>1.04</td>
</tr>
<tr>
<td>$\rho_g$</td>
<td>0.00–0.11</td>
<td>0.03–0.29</td>
<td>-0.13</td>
<td>-0.11</td>
<td>-0.11</td>
</tr>
<tr>
<td>$\sigma_g$</td>
<td>2.13–3.06</td>
<td>0.47–1.20</td>
<td>2.89</td>
<td>5.33</td>
<td>6.20</td>
</tr>
<tr>
<td>$\rho_a$</td>
<td>0.95</td>
<td>0.97</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma_a$</td>
<td>0.17–0.54</td>
<td>0.63–0.78</td>
<td>0.68</td>
<td>0.73</td>
<td>0.58</td>
</tr>
</tbody>
</table>
Finally, column 12 of Table 3 shows the nature of technology shock processes for India. To have an account of transitory and trend growth shock in the Indian TFP series, we decompose the series in permanent and transitory components using Kalman filter. First, we estimate (TFP) for India. The aggregate production function, representing the production sector in our model outlined in the next section, is defined following [Aguiar and Gopinath (2007)] as

\[
Y_t = e^{a_t} K_t^{1-\alpha} (\Gamma_t)^{\alpha}
\]

where \( K_t \) is the aggregate stock of capital and \( \alpha \in (0, 1) \) denotes labour’s share of output. We assume that households supply unit labour inelastically.

The parameters \( a_t \) and \( \Gamma_t \) represents productivity processes. The two productivity processes are characterised by different stochastic properties. The parameter \( a_t \) captures a transitory movement in productivity and is characterised by the following AR(1) process:

\[
a_t = \rho_a a_{t-1} + \epsilon^a_t; \quad |\rho_a| < 1, \quad \epsilon^a_t \sim N(0, \sigma_a^2)
\]

The parameter \( \Gamma_t \) represents the cumulative product of growth shocks as follows:

\[
\ln \left( \frac{g_t}{\mu_g} \right) = \rho_g \ln \left( \frac{g_{t-1}}{\mu_g} \right) + \epsilon^g_t; \quad |\rho_g| < 1, \quad \epsilon^g_t \sim N(0, \sigma_g^2)
\]

where \( \mu_g - 1 \) is the long-run mean trend growth rate. The two different productivity processes are assumed to distinguish the shock process to the level of productivity \( a_t \) and the growth rate of productivity \( g_t \). The growth shocks are incorporated in a labour augmenting way to ensure the existence of a steady state where all variables grow at the rate \( \mu_g \), and for the tractability of analysis of cyclical properties of the model economy. In this analysis, the cyclical component of a variable \( X_t \), i.e., the deviation of the variable from its trend path is defined as \( x_t = \frac{X_t}{\Gamma_{t-1}} \).

The Solow residual from the aggregate production function captures productivity processes that contains a transitory and a permanent component:

\[
sr_t = a_t + \alpha \ln \Gamma_t = \ln Y_t - (1 - \alpha) \ln K_t
\]

Since, we assume that the households supply unit labour inelastically and total mass of households is normalised to one, equation 4 measures the Solow
residual in terms of per capita output and capital stock. We estimate the Solow residual for India. GDP at factor cost and net fixed capital stock, both in 2004-05 constant prices, proxy for output and capital stock respectively. The data on GDP and net fixed capital stock are sourced from National Accounts Statistics. The labour force data are sourced from World Bank. The value of labour share is set to 0.7 from Verma (2008). Given the availability of data on labour force and capital stock, our Solow residual series spans 1980-2009.

We estimate transitory and permanent components in the Solow residual series for India using Kalman filter. The underlying model is the following: the Solow residual series $sr_t$ is a sum of a random walk or trend component $T_t$ and a transitory or cyclical component $C_t$:

\[
sr_t = T_t + C_t + V_t; \quad V_t \sim N(0, \sigma_V^2)
\]

\[
T_t = d + T_{t-1} + W_{1t}; \quad W_{1t} \sim N(0, \sigma_{W1}^2)
\]

\[
C_t = \rho_c C_{t-1} + W_{2t}; \quad |\rho_c| < 1; \quad W_{2t} \sim N(0, \sigma_{W2}^2)
\]  

where $V_t$ represents measurement error. This Trend-Cycle model in equation 5 can be represented in state-space form as:

\[
sr_t = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} T_t \\ C_t \end{bmatrix} + V_t
\]

\[
\begin{bmatrix} T_t \\ C_t \end{bmatrix} = \begin{bmatrix} d \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & \rho_c \end{bmatrix} \begin{bmatrix} T_{t-1} \\ C_{t-1} \end{bmatrix} + \begin{bmatrix} W_{1t} \\ W_{2t} \end{bmatrix}
\]  

The first expression in equation 6 represents the observation equation in terms of the unobserved states. The second equation represents the transition dynamics of the state variables. Figure 2 depicts Kalman filtered trend growth rate and cyclical components of Solow residual for India.
Decomposition of Indian TFP in permanent and transitory components show that shocks to trend growth is a major source of fluctuations in Indian business cycle. The Kalman filtered estimate of $\sigma_W^2 = 0.32$ provides a measure of transitory shock $\sigma_a$, and the estimate of $\rho_c = 0.84$ gives the degree of persistence in transitory component of TFP. Next, we fit an AR(1) model to the growth rate of the estimated permanent component of TFP. The persistence in the trend growth $\rho_g$ is found to be 0.27, while the estimate of $\sigma_g$ is 1.59. The value of $\sigma_g$ compared $\sigma_a$ indicates that the shock to trend growth rate is substantially higher than the transitory shock. These estimates are shown in Table 3 along with output and consumption volatilities during the period spanning our TFP series.

4 Credit constraints and consumption volatility: Theoretical framework

The theoretical literature on finance and macroeconomic volatility explores how financial integration and financial development affect output and consumption volatility through the channel of firms and households (Bernanke and Gertler, 1989; Greenwald and Stiglitz, 1993; Iyigun and Owen, 2004; Buch et al., 2005; Leblebicioglu, 2009; Aghion et al., 2010). The effect of financial integration on macroeconomic volatility dominates the literature. A limited strand of literature explores the role of domestic financial development in determining the pattern of macroeconomic fluctuations and the bulk of it focuses on the channel of firms (Bernanke and Gertler, 1989; Greenwald and Stiglitz, 1993; Aghion et al., 2010).

The early literature predicts that financial development reduces macroeconomic fluctuations (Bernanke and Gertler, 1989; Greenwald and Stiglitz, 1993). More recent literature suggests that the nature of relationship between financial development and macroeconomic volatility can be non-linear and may depend on several factors, such as the composition of short-term and long-term investments in the economy (Aghion et al., 2010).

4.1 The model

Consider a closed economy is populated by a continuum of infinitely lived households and firms, both of measure unity. There exist a fraction $\lambda$ of households with no access to banking or other instruments to save. These
consumers, who may be referred to as non-Ricardian households, are liquidity-
constrained and unable to save or borrow to smooth consumption. They have no assets and spend all their current disposable labour income on consumption in each period.

Labour supply is inelastic as no labour-leisure choice is made by the representative household. Emerging economies are characterised by large size of informal employment where average hours of work are found to be higher than that in the formal sector employment (Blunch et al. 2001; International Labour Organization 2012). For instance, studies found that informal sector workers worked on average 15 hours more than their counterparts in the formal sector (Blunch et al. 2001). Hence, in an emerging economy set up, it is reasonable to assume that households allocate their available labour-time to production as much as possible. We assume that the representative household supplies one unit of labour inelastically.

Both Ricardian and liquidity-constrained households have identical preferences defined over a single commodity,

$$U(C_i^t) = \ln(C_i^t) \quad i = R, L$$

where $C_i^t$ denotes total consumption of the household of type $i$. Ricardian households are indexed as $R$ and liquidity constrained households as $L$.

A Ricardian household maximises discounted stream of utility

$$V_t = E_t \sum_{t=0}^{\infty} \beta^t \log(C_t^R),$$

subject to the following budget constraint,

$$C_t^R + I_t^R = R_tK_t^R + W_t,$$

In India, more than 90% of the workforce and about 50% of the national product are accounted for by the informal economy (Report of the Committee on Unorganised Sector Statistics 2012). According to NSS Report (2004-05), of the total workers, 82% in the rural areas and 72% in the urban areas are engaged in informal sector. In terms of absolute numbers, out of the total 465 million people employed in the formal and informal sectors, only 28 million people (6% of the total employment) are employed in the formal sector, while 437 million workers (94% of the total employment) are in the informal sector (National Sample Survey Organisation 2009-10). Data on hours worked are not officially published in India. The officially published employment data captures the employment scenario in the formal sector which constitutes only 6% of the total employment.
where $\beta \in (0, 1)$ denotes the subjective discount factor. Here $C^R_t$ is total consumption of the Ricardian household in period $t$. The variables $I^R_t$ and $K^R_t$ denote investment and capital stock of the household respectively. The economy-wide return to capital and wage rate are given by $R_t$ and $W_t$. In each period the Ricardian household divides her disposable income comprised of wage and rental income into consumption and savings.

The stock of capital of the representative Ricardian household evolves via the following law of motion,

$$K^R_{t+1} = (1 - \delta)K^R_t + I^R_t - \frac{\phi}{2}\left(\frac{K^R_{t+1}}{K^R_t} - \mu_g\right)^2 K^R_t \tag{10}$$

The investment is subject to quadratic capital adjustment cost as in Aguiar and Gopinath (2007).

Households who do not have access to financial services cannot save or borrow. Their behaviour is thus different from that of Ricardian consumers. Liquidity constrained households maximise instantaneous utility $\log C^L_t$ subject to the following budget constraint in each period,

$$C^L_t = W_t, \tag{11}$$

where $C^L_t$ is total consumption of the liquidity constrained household in period $t$. In each period, a liquidity constrained household consumes its entire disposable income comprised of wage income.

The aggregate consumption is the weighted average of consumption by the liquidity-constrained households and the Ricardian households. The weights are the share of each type of households in the population.

$$C_t = \lambda C^L_t + (1 - \lambda)C^R_t. \tag{12}$$

The aggregate capital stock and investment are respectively the following

$$K_t = (1 - \lambda)K^R_t, \quad I_t = (1 - \lambda)I^R_t, \tag{13}$$

A representative firm produces a homogeneous good, by hiring one unit of labour from households and combining it with capital. The aggregate output is produced by Cobb Douglas technology that uses capital and unit labour as inputs,

$$Y_t = e^{\alpha\gamma}(1 - \lambda)K^R_t^{1-\alpha} = \Gamma^\alpha, \tag{14}$$

13
where $\alpha \in (0, 1)$ represents labour’s share of output and $e^a_t$ denotes the transitory component of total factor productivity. Here $\Gamma_t$ is the permanent component of productivity. The two productivity processes are characterised by the following stochastic properties: total factor productivity evolves according to an AR(1) process as follows:

$$a_t = \rho_a a_{t-1} + \epsilon^a_t$$  \hspace{1cm} (15)

with $|\rho_a| < 1$ and $\epsilon^a_t$ represents iid draws from a normal distribution with zero mean and standard deviation $\sigma_a$.

Following Aguiar and Gopinath (2007), the growth rate of labour productivity $\Gamma_t$ is defined as

$$\Gamma_t = g_t \Gamma_{t-1}$$  \hspace{1cm} (16)

The growth rate of labour productivity $g_t$ follows an AR(1) process of the form:

$$\ln \left( \frac{g_t}{\mu_g} \right) = \rho_g \ln \left( \frac{g_{t-1}}{\mu_g} \right) + \epsilon^g_t; \epsilon^g_t \sim N(0, \sigma^2_g)$$  \hspace{1cm} (17)

The resource constraint of the economy is given by

$$C_t + I_t = Y_t$$  \hspace{1cm} (18)

In a closed economy, total output is allocated between total consumption and investment as indicated by equation (18).

Since the realisation of $g$ permanently influences $\Gamma$, output is non-stationary with a stochastic trend. We detrend output, consumption, investment and capital stock by normalising these variables with respect to the trend productivity through period $t-1$. For any variable $X$, its detrended counterpart is defined as $x_t = \frac{X_t}{\Gamma_{t-1}}$.

With the initial capital stock $K_0$, the competitive equilibrium is defined as a set of prices and quantities $(R_t, W_t, y_t, c_t, c_t^R, c_t^I, i_t, k_t)$, given the sequence of shocks to TFP and labour productivity growth, that solves the maximisation problem of the household, optimisation by the firms and satisfies the resource constraint of the economy.

### 4.2 Predictions

After normalisation of the variables by labour productivity in the previous period, the system of equations driving the dynamics of the model economy
become

\[ 1 = \beta E_{t-1} \left[ \Omega_t \frac{c_t^R}{c_t^R g_t} \right] \]

\[ \Omega_t = (1 - \alpha) e^{\alpha a t} (1 - \lambda)^{1-\alpha} (k_t^R)^{-\alpha} g_t + (1 - \delta), \]

\[ c_t^R = \frac{(1 - \alpha \lambda)}{1 - \lambda} e^{\alpha a t} [(1 - \lambda) k_t^R]^{-\alpha} g_t + (1 - \delta) k_t^R \]

\[ -g_t k_{t+1} - \frac{\phi}{2} \left( \frac{k_{t+1}^R g_t}{k_t} - \mu_g \right)^2 k_t, \]

\[ a_t = \rho_a a_{t-1} + \epsilon_a^t, \]

\[ \ln \left( \frac{g_t}{\mu_g} \right) = \rho_g \ln \left( \frac{g_{t-1}}{\mu_g} \right) + \epsilon_g^t. \]

(19)

The first equation in the system of equations (19) describes intertemporal allocation of consumption by the Ricardian consumers where \( \Omega_t \) is the gross return to capital. The third equation pertains to the resource constraint of the economy, after taking into account the consumption of liquidity-constrained households as in equation (11), total consumption in equation (12), dynamics of capital accumulation by the Ricardian households in equation (10), stock of capital and investment in the economy given in equation (13), and making use of the fact that \( W_t = \alpha e^{\alpha a t} [(1 - \lambda) k_t^R]^{1-\alpha} g_t. \)

After log-linearising the system of equations (19) and given the total consumption of the economy as in equation (12), and making use of the equation (11) and the fact that \( W_t = \alpha Y_t \) implying \( \tilde{c}_t^L = \tilde{g}_t \), one can arrive at the volatility of consumption relative to output as,

\[ \frac{\sigma_{\tilde{c}}^2}{\sigma_{\tilde{g}}^2} = \left( \frac{\tilde{c}_t^R}{\tilde{c}^*} \right)^2 (1 - \lambda)^2 \sigma_R^2 + \left( \frac{\tilde{c}_t^L}{\tilde{c}^*} \right)^2 \lambda^2. \]

(20)

Here the fluctuations in a Ricardian household’s consumption and that in total output are respectively,

\[ \sigma_{\tilde{c}^R}^2 = \left[ \frac{a_2^2 b_1^2}{1 - a_1^2} + b_2^2 \right] \sigma_a^2 + \left[ \frac{a_2^2 d_1^2}{1 - a_1^2} + d_2^2 \right] \sigma_g^2, \]

\[ \sigma_{\tilde{g}}^2 = \left[ 1 + \frac{(1 - \alpha)^2 b_1^2}{1 - a_1^2} \right] \sigma_a^2 + \left[ \alpha^2 + \frac{(1 - \alpha)^2 d_1^2}{1 - a_1^2} \right] \sigma_g^2. \]

Appendix B describes the solution method in details.

The effects of transitory and permanent income shocks on the volatility of consumption in the economy relative to volatility of output can be summarised as follows.
Proposition 1  With everything else remaining unchanged,
(i) Volatility of consumption of a liquidity constrained household relative to output volatility is always unity, i.e., \( \frac{\sigma_{\tilde{c}_{L}}}{\sigma_{\tilde{y}}} = 1 \), when \( \sigma_{\epsilon_{a}} > 0; \sigma_{\epsilon_{g}} > 0 \).
(ii) Due to a transitory shock in income, both volatility of consumption of a Ricardian household relative to output volatility and the volatility of total consumption relative to output volatility are lower than 1, irrespective of the share of liquidity constrained households in the population, i.e., \( \frac{\sigma_{\tilde{c}_{R}}}{\sigma_{\tilde{y}}} < 1 \) and \( \frac{\sigma_{\tilde{c}}}{\sigma_{\tilde{y}}} < 1 \) for \( \lambda \in [0, 1) \), when \( \sigma_{\epsilon_{a}} > 0; \sigma_{\epsilon_{g}} = 0 \).
(iii) Due to a shock to the trend growth of income, volatility of consumption of a Ricardian household relative to volatility of output always exceeds 1, irrespective of the share of liquidity constrained households in the economy, while the volatility of total consumption relative to output volatility depends on the share of liquidity constrained households in the economy, i.e., \( \frac{\sigma_{\tilde{c}_{R}}}{\sigma_{\tilde{y}}} > 1 \), and \( \frac{\sigma_{\tilde{c}}}{\sigma_{\tilde{y}}} \geq 1, \) for \( \lambda \in [0, 1) \), when \( \sigma_{\epsilon_{a}} = 0; \sigma_{\epsilon_{g}} > 0 \).
(iv) In the presence of shock to the trend growth rate, both volatility of consumption of a Ricardian household relative to output volatility and the volatility of total consumption relative to volatility of output increases when the share of liquidity-constrained households in the economy decreases, i.e., \( \frac{\partial}{\partial \lambda} \left( \frac{\sigma_{\tilde{c}_{R}}}{\sigma_{\tilde{y}}} \right) < 0 \), and \( \frac{\partial}{\partial \lambda} \left( \frac{\sigma_{\tilde{c}}}{\sigma_{\tilde{y}}} \right) < 0 \), for \( \lambda \in [0, 1) \), when \( \sigma_{\epsilon_{a}} = 0; \sigma_{\epsilon_{g}} > 0 \).

The proof of the Proposition 1 is presented in Appendix B in details.

Liquidity constrained households who have no access to savings instruments can respond to any change in income by changing consumption by the amount of changed income. Hence volatility of consumption of a liquidity constrained household relative to output volatility is always one irrespective of the nature of shock.

In response to a transitory income shock, a Ricardian household smooths consumption by re-allocating changed income between consumption and savings. Hence consumption fluctuates by a lesser amount compared to income fluctuation. Hence consumption volatility of a Ricardian household relative to output volatility is always less than one irrespective of the level of financial development. Relative volatility of total consumption when total consumption is a weighted average of the relative consumption volatility of a Ricardian household and that of a liquidity constrained household is also less than one in all states of financial development.

Ricardian households perceive a rise in income in the future following a per-
manent income shock. They respond to it by raising current consumption more than the rise in current income by borrowing against future income or reducing current savings. Thus relative volatility of consumption of a Ricardian household with respect to output volatility is greater than one. Relative volatility of total consumption when total consumption is a weighted average of the relative consumption volatility of a Ricardian household and that of a liquidity constrained household may be smaller or higher than 1 depending on the size of $\lambda$.

Financial development reduces the share of liquidity constrained households in the economy and hence allows more people to respond to the permanent income shock by raising current consumption more than the rise in current income. As a result, volatility of total consumption relative to output volatility increases with financial development.

Combining these observations, the main theoretical prediction of our model can be stated as follows.

**Main prediction:** Other things unchanged, under the occurrence of permanent income shock, financial development leads to a rise in the volatility of consumption in the economy relative to output volatility.\footnote{It follows from the implications of the main prediction of the model that in response to a negative permanent income shock, Ricardian households reduce current consumption by more than the decline in current income and raise investment in order to smooth consumption over the lifetime. Financial development will allow more people to respond to the negative income shock by reducing current consumption more than the fall in income. Volatility of total consumption relative to output volatility, thus, increases with financial development under negative trend growth shocks as well.}

We test this prediction by calibrating the model economy to Indian data. We test our hypothesis for an emerging economy where relative consumption volatility shows an increase after witnessing of financial sector development.

## 5 Case study

### 5.1 Evidence for India

We calibrate the model for India, an emerging economy which has witnessed financial sector reform.\footnote{Ang (2011) finds that financial liberalisation increases fluctuations in consumption in India during 1950-2005. Also, relative} Ang (2011) finds that financial liberalisation increases fluctuations in consumption in India during 1950-2005. Also, relative
Figure 3 Financial development in India

This figure shows the behaviour of some financial development indicators in India. The ratio of total number of bank accounts in the economy to total population, the number of bank branches in 100,000 population, bank deposit to GDP ratio and private credit to GDP are all seen to rise. The dashed lines show the mean values before and after financial reforms.
Table 4 Business cycle stylised facts for the Indian economy in the pre and post reform period

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Std. dev. Rel. dev. Cont. dev.</td>
<td>First ord. Std. dev. Rel. dev. Cont. dev.</td>
</tr>
<tr>
<td>Real GDP</td>
<td>2.25</td>
<td>1.00</td>
</tr>
<tr>
<td>Pvt. Cons.</td>
<td>1.86</td>
<td>0.83</td>
</tr>
<tr>
<td>Investment</td>
<td>5.26</td>
<td>2.34</td>
</tr>
</tbody>
</table>

to income volatility, consumption volatility in India increased after reform (Ghate et al., 2013).

India has witnessed development of its domestic financial sector in the post reform period, while remaining fairly closed in terms of capital account openness even after the reform. Thus India serves as an example of an emerging economy, with a low level of financial integration and a moderate expansion of domestic financial services. Figure 3 shows the expansion of financial services in India from the pre to post-reform periods. Interestingly, the country witnessed a small decline in banking services before witnessing a sharp increase. This period is included in the post reform sample to achieve reasonable sample size.

We simulate this model for the pre and post-reform periods, keeping all deep parameters, except the share of non-Ricardian households the same for both periods. Expansion of the financial services is captured by a lower value of the share of liquidity-constrained households in the post reform period. In this way we seek to identify one of the key factors which may explain the differences in relative consumption volatility between pre and post financial reform periods. We simulate the model for two different values of the share of liquidity constrained households and compare the simulated business cycle moments with business cycle stylised facts observed in pre and post reform India.

We calculate key business cycle moments for per capita output, consumption and investment at annual frequency. Output, consumption and investment are measured by real GDP at factor cost, private consumption expenditure and gross fixed capital formation for the period 1951-2010. To examine the transition in the business cycle stylised facts, the sample is divided into pre (1951-1991), and post reform periods (1992-2010). Key business cycle moments are obtained from the HP-filtered cyclical components of per capita output, consumption and investment.

The change in business cycle facts for the Indian economy from 1951-2009 are
Figure 4 Trend in relative consumption volatility

This figure shows five year rolling relative consumption volatility in India during 1956-2009.

depicted in Table 4. Per capita Real GDP has become less volatile in the post-reform period in India. The level of volatility is still high and comparable to emerging economies. The absolute per capita consumption volatility as well as the relative consumption volatility with respect to output increased in the post-reform period. Per capita investment volatility show a small decline in the post-reform period, while volatility in investment relative to output volatility has increased following reform. Contemporaneous correlation of consumption and investment with output has increased in the post-reform period. No significant persistence in the output and consumption cycle is seen in the pre-reform period. In the post-reform period, output and consumption cycle are observed to have higher persistence. Persistence in the investment cycle rises in the post-reform period.

There has been a sharp increase in access to finance after reforms. The ratio of bank account in total population was merely 20% in 1980, it has jumped to above 70% in 2010, except for a period of decline in the trend during 1990-2005. Similarly bank branches per 100,000 population in 2010 are more than double the value in 1970.

As seen in Table 4, relative consumption volatility in India has risen from 0.83 during 1951-1991 to 1.04 during 1992-2012. Thus after improved access to savings instruments and credit, fluctuations in consumption relative to fluctuations in income has increased.

Figure 4 shows the trend in one of the key variables of our analysis, namely, relative consumption volatility. The figure shows that the mean of relative
Table 5 Benchmark parameter values

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount factor</td>
<td>$\beta$ 0.968</td>
</tr>
<tr>
<td>Rate of Depreciation (%)</td>
<td>$\delta$ 5.000</td>
</tr>
<tr>
<td>Share of labour</td>
<td>$\alpha$ 0.700</td>
</tr>
<tr>
<td>Adjustment cost parameter</td>
<td>$\phi$ 2.820</td>
</tr>
<tr>
<td>Mean trend growth rate of labour productivity (%)</td>
<td>$\mu_g - 1$ 2.790</td>
</tr>
<tr>
<td>Persistence in transitory component of technology</td>
<td>$\rho$ 0.713</td>
</tr>
<tr>
<td>Volatility in transitory component of technology (%)</td>
<td>$\sigma_a$ 0.320</td>
</tr>
<tr>
<td>Persistence in growth of permanent component of technology</td>
<td>$\rho_g$ 0.266</td>
</tr>
<tr>
<td>Volatility of shock to permanent component of technology (%)</td>
<td>$\sigma_g$ 1.590</td>
</tr>
</tbody>
</table>

consumption volatility has increased in the post reform period.

5.2 Calibration

Table 5 summarises the benchmark parameter values used in our calibration exercise. The access of households to banking is captured by the number of bank accounts to population. Hence the proxy for $\lambda$, i.e., the share of liquidity constrained households is derived from this ratio. The number of bank accounts to population ratios in 1980 and 2010 are used to calibrate the share of liquidity constrained households in the pre and post reform periods. In 1980 21.4% of the population had access to banking. Thus the share of households without access to finance, i.e., $\lambda$ is set to 0.786 in the pre reform period. In 2010 66.9% of the population had access to banking services. The value of $\lambda$ is $1 - 0.669 = 0.331$ in the post reform period.

We follow the existing literature in choosing some of the other parameter values. A period is a year. The share of labour $\alpha$ for India is 0.7 as in Verma (2008), while the rate of depreciation is 5% as in Virmani (2004).

Next, we calibrate the annual discount rate using annual data of real interest rate for India sourced from the World Bank. The real interest rate series reported in this database is the lending interest rate adjusted for inflation as measured by the GDP deflater. We estimate the trend real interest rate using the Hodrick-Prescott filter. The average value of the trend real interest rate during the sample period of 1980-2012 is $\bar{R} = 6.16\%$. The Euler equation in steady state becomes $\mu_g = \beta (1 + \bar{R})$, where $\mu_g - 1$ is the average trend growth of productivity process, and $\beta$ is the annual discount factor. We estimate $\mu_g - 1$ using Kalman filtration of Solow residual series for India.\(^5\)

\(^5\)The details of the estimation procedure and results are outlined in section 3.
Table 6 Business cycle volatilities from the simulated model

This table presents absolute and relative business cycle volatilities from the simulated model for the pre and post-reform period.

<table>
<thead>
<tr>
<th></th>
<th>Std. dev.</th>
<th>C</th>
<th>I</th>
<th>Rel. std. dev.</th>
<th>C</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Data</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pre-reform</td>
<td>2.25</td>
<td>1.86</td>
<td>5.26</td>
<td>0.83</td>
<td>2.34</td>
<td></td>
</tr>
<tr>
<td>Post-reform</td>
<td>1.93</td>
<td>1.99</td>
<td>5.18</td>
<td>1.04</td>
<td>2.69</td>
<td></td>
</tr>
<tr>
<td><strong>Model</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pre-reform</td>
<td>1.92</td>
<td>1.97</td>
<td>4.46</td>
<td>1.03</td>
<td>2.32</td>
<td></td>
</tr>
<tr>
<td>Post-reform</td>
<td>1.91</td>
<td>2.16</td>
<td>3.53</td>
<td>1.13</td>
<td>1.85</td>
<td></td>
</tr>
</tbody>
</table>

The estimated value of $\mu_g - 1$ is 2.79%. It then follows from the Euler equation that annual discount factor for India is $\beta = \frac{\mu_g}{1+\bar{R}} = \frac{1.0279}{1.0616} = 0.968$.

The estimated shock processes in the transitory and the growth rate of permanent components of Solow residual for India are sourced from Table 3. The parameter for capital adjustment cost $\phi$ is set to 2.82 from Aguiar and Gopinath (2007).

5.3 Effect of financial development on relative consumption volatility

Our model predicts that a decline in the share of liquidity-constrained households in the population would allow more people to respond to permanent income shocks. They can increase current consumption more than the rise in current income. This is predicted to result in a rise in the relative consumption volatility.

Table 6 presents our main findings. Relative consumption volatility shows a rise in the post reform period. This result supports our key prediction. Since financial development allows more people to access savings instruments, when households perceive a permanent income shock which raises both current and future income, more people can respond to the shock by reducing current savings and raising current consumption more than the rise in current income. As a result of financial development, the volatility of consumption relative to volatility of output rises.

Our model also replicates the pattern of changes in absolute consumption volatility successfully. The model also captures a decline in the absolute output volatility in the post reform period as observed in the data. However,
Table 7 Business cycle correlation and persistence from the simulated model

This table presents absolute and relative business cycle volatilities from the simulated model for the pre and post-reform period.

<table>
<thead>
<tr>
<th></th>
<th>Correlation</th>
<th>Auto-correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>C</td>
<td>T</td>
</tr>
<tr>
<td>Data</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pre-reform</td>
<td>0.70</td>
<td>0.19</td>
</tr>
<tr>
<td>Post-reform</td>
<td>0.92</td>
<td>0.76</td>
</tr>
<tr>
<td>Model</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pre-reform</td>
<td>0.99</td>
<td>0.22</td>
</tr>
<tr>
<td>Post-reform</td>
<td>0.97</td>
<td>0.24</td>
</tr>
</tbody>
</table>

in terms of magnitude, the change in the output volatility is not substantial. With financial inclusion, more people can save and hence, investment volatility declines. Our model shows a fall in the absolute volatility in investment in the post-reform period, as observed empirically. However, unlike the trend shown in the data, the simulated relative investment volatility declines in the post reform period.

Table 7 compares the simulated correlation of consumption and investment cycles with the output cycle and their persistence with the empirical counterparts. The model shows a rise in the correlation of investment with output, as in the data. However the magnitude of the rise is small, compared to the trend shown by the data. The simulated correlation of consumption cycle with the output cycle shows a marginal decline after reform.

The pattern of model simulated persistence in output and consumption cycles matches broadly with the pattern observed in the data. However, the performance of the model is not satisfactory in terms of matching the persistence in the investment cycle. Finally, Figure 5 shows that our model replicates the cyclical pattern in output, consumption and investment fairly well.

5.4 Sensitivity to the measure of financial development

In the above analysis, we measured financial development as the share of the population with bank accounts. As a robustness check we now use another measure of financial development, the bank deposit to GDP ratio to obtain the fraction of liquidity constrained households in the economy. By this measure, $\lambda = 0.687$ in the pre-reform period and 0.305 in the post liberalisation period.
Figure 5 Actual and simulated cycles

This figure compares cyclical movements in per capita GDP, consumption expenditure and investment in the pre and post-reform periods with simulated output, consumption and investment cycles for these two periods.
Table 8 Sensitivity analysis with respect to the financial development parameter

This table presents business cycle moments from the simulated model for the pre and post-reform period using an alternative measure of $\lambda$. The measure used in this analysis is based on the deposit to GDP ratio.

<table>
<thead>
<tr>
<th></th>
<th>Std. dev.</th>
<th>Rel. std. dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Y</td>
<td>C</td>
</tr>
<tr>
<td>Data</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pre-reform</td>
<td>2.25</td>
<td>1.86</td>
</tr>
<tr>
<td>Post-reform</td>
<td>1.93</td>
<td>1.99</td>
</tr>
<tr>
<td>Model</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pre-reform</td>
<td>1.92</td>
<td>2.00</td>
</tr>
<tr>
<td>Post-reform</td>
<td>1.91</td>
<td>2.18</td>
</tr>
</tbody>
</table>

Table 9 Sensitivity analysis with respect to the financial development parameter

This table shows that business cycle moments from the simulated model for the pre and post-reform period using the alternative measure of $\lambda$ based on deposit to GDP ratio are broadly unchanged.

<table>
<thead>
<tr>
<th></th>
<th>Correlation</th>
<th>Auto-correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>C</td>
<td>I</td>
</tr>
<tr>
<td>Data</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pre-reform</td>
<td>0.70</td>
<td>0.19</td>
</tr>
<tr>
<td>Post-reform</td>
<td>0.92</td>
<td>0.76</td>
</tr>
<tr>
<td>Model</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pre-reform</td>
<td>0.99</td>
<td>0.23</td>
</tr>
<tr>
<td>Post-reform</td>
<td>0.96</td>
<td>0.24</td>
</tr>
</tbody>
</table>
Table 8 and 9 shows that key moments from the business cycle model for the pre and post-reform periods based on an alternative measure of $\lambda$ are similar to those of the benchmark model.

6 Financial development, permanent income shock and relative consumption volatility: implications for a small open economy

Along with domestic financial deepening, opening up of the capital account, or financial liberalisation has been a major component of the spectrum of reforms in emerging economies in the last two decades. This section explores the implications of financial deepening for the aggregate consumption fluctuations in an open economy framework.

We assume that financial transactions by Ricardian households take place through an internationally traded, one-period risk-free bond as in Aguiar and Gopinath (2007). The budget constraint of the Ricardian households is modified for the open economy framework as

$$C_t^R + I_t^R + B_t^R - \frac{B_{t+1}^R}{1 + R_t} + = R_t^K K_t^R + W_t$$

(21)

Here, the level of debt due in period $t$ held by a Ricardian household is denoted by $B_t^R$ and $R_t$ is the time $t$ interest rate payable for the debt due in period $t + 1$. The economy-wide return to physical capital and wage rate are given by $R_t^K$. Access to international financial markets is assumed to be imperfect. The interest rate is subject to a premium associated to the riskiness of investing in emerging economies. This premium depends on the level of outstanding debt, taking the form used in Schmitt-Grohe and Uribe (2003).

$$R_t = R^* + \psi (e^{\frac{B_t - \bar{b}}{R_t}} - 1).$$

(22)

Here the variable $R^*$ is the world interest rate exogenously given to the small open home country. The variable $\bar{b}$ denotes the steady state level of total debt, and $\psi (\psi > 0)$ is the elasticity of interest rate to changes in the indebtedness of the economy. The total debt of the economy $B_t$ is exogenously given to the representative agent who does not internalise the premium payable on the foreign interest rate determined by the indebtedness of the economy. However, in equilibrium, total foreign debt of the economy coincides with the amount of debt acquired by all the representative agents.
of the Ricardian type. Given the fraction of Ricardian households in the economy equal to $1 - \lambda$, the total debt in the economy amounts to $B_t = (1 - \lambda)B^R_t$, while the long run total debt is $\bar{b} = (1 - \lambda)\bar{b}^R$.

The resource constraint equation for the open economy is modified as follows:

$$C_t + I_t + TB_t = Y_t$$  \hspace{1cm} (23)

where the trade balance $TB_t$ is financed by the net flows of capital,

$$TB_t = B_t - \frac{B_{t+1}}{1 + R_t}$$  \hspace{1cm} (24)

In an economy which is open on both trade and financial fronts, imports and total domestic output net of exports is allocated between total consumption and investment, where the difference between exports and imports are balanced by the financial flows as indicated by equations (23) and (24). The rest of the framework, such as optimisation problem of the Ricardian and the liquidity constrained households, firm’s profit maximisation behaviour and the permanent and transitory shock structures remain similar as in the closed economy framework. By normalising the variables with respect to the permanent component of productivity at period $t - 1$, the detrended system of equations are obtained. Appendix C contains the detrended system of equations pertaining to the open economy.

In order to calibrate the open economy, we choose a value of the interest rate elasticity of indebtedness of 0.001 as in Aguiar and Gopinath (2007). The steady state level of debt to GDP ratios for the pre and post reform periods are set to the average values of the external debt to GDP ratios in 1971-1991 and 1992-2012 respectively. The respective values are 16.30% and 21.39%.\footnote{The annual series of external debt are sourced from WDI. The data spans from 1971-2012 and are in current US $. The GDP data, also in current US $, are sourced from WDI.}

The value of the risk-free world interest rate is set to satisfy the condition that $\beta(1 + R^*) = \mu_g$, where $\mu_g - 1$ is the mean growth rate of the permanent component of TFP. The value of this parameter is set to 2.79%, the value of the average growth rate of the permanent component of TFP as estimated in Section 3. The rest of the parameter values remain same as in the closed economy case.

Table 10 presents India’s business cycle stylised facts for various macroeconomic indicators including government’s expenditure to GDP ratio and the trade balance to GDP ratio for the pre and post reform periods. The table
Table 10 Business cycle stylised facts for the Indian economy in the pre and post reform period

<table>
<thead>
<tr>
<th></th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Real GDP</td>
<td>2.25</td>
<td>1.00</td>
</tr>
<tr>
<td>Pvt. Cons.</td>
<td>1.86</td>
<td>0.83</td>
</tr>
<tr>
<td>Investment</td>
<td>5.26</td>
<td>2.34</td>
</tr>
<tr>
<td>Gov. exp./GDP</td>
<td>6.19</td>
<td>2.75</td>
</tr>
<tr>
<td>Trade bal./GDP</td>
<td>0.90</td>
<td>0.40</td>
</tr>
</tbody>
</table>

Table 11 Average ratios of government expenditure to GDP and trade balance to GDP

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean Govt. exp/GDP</td>
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<td>16.52</td>
</tr>
<tr>
<td>Mean Trade bal./GDP</td>
<td>1.99</td>
<td>3.48</td>
</tr>
</tbody>
</table>

shows that the absolute volatility in government expenditure to GDP ratio declines in the post reform period, where as the relative volatility has increased marginally. The volatilities, both absolute and relative, in net exports to GDP ratio have increased in the post reform period. The average net exports to GDP ratio and the average government expenditure to GDP ratios are reported in Table [11]. Mean net exports to GDP ratio shows an increase in the post reform period, where as the average government expenditure to GDP ratio declines marginally after reform.

Tables [12] and [13] compare empirical and simulated business cycle moments for the open economy in the pre and post reform periods. The open economy version of the model is able to replicate most of the patterns in the changes in stylised facts from the pre to post-reform periods in India. As we observe in the data, the model-simulated absolute volatilities in consumption and trade balance to GDP ratio have increased in the post reform period, while that of investment has decreased. However, unlike in the data, the volatility of output in the model shows a rise in the post-reform period and the absolute volatility in the trade balance to GDP ratio exceeds output volatility.

So far as the relative volatilities are concerned, volatilities in consumption and trade balance to GDP ratio, relative to output volatility rise, reflecting trends observed in the data. However, unlike the pattern observed empirically, the relative volatility of investment falls. The relative volatility of investment resembles the pattern observed in the closed economy framework.
Table 12 Business cycle volatilities from the simulated model

This table presents absolute and relative business cycle volatilities from the simulated model for the pre and post-reform period.

<table>
<thead>
<tr>
<th></th>
<th>Std. dev.</th>
<th>Rel. std. dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Y</td>
<td>C</td>
</tr>
<tr>
<td>Data</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pre-reform</td>
<td>2.17</td>
<td>1.86</td>
</tr>
<tr>
<td>Post-reform</td>
<td>1.94</td>
<td>1.99</td>
</tr>
<tr>
<td>Model</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pre-reform</td>
<td>1.48</td>
<td>2.14</td>
</tr>
<tr>
<td>Post-reform</td>
<td>1.51</td>
<td>2.94</td>
</tr>
</tbody>
</table>

Table 13 Business cycle correlation and persistence from the simulated model

This table presents absolute and relative business cycle volatilities from the simulated model for the pre and post-reform period.

<table>
<thead>
<tr>
<th></th>
<th>Correlation</th>
<th>Auto-correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Correlation</td>
<td>Auto-correlation</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>I</td>
</tr>
<tr>
<td>Data</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pre-reform</td>
<td>0.71</td>
<td>0.19</td>
</tr>
<tr>
<td>Post-reform</td>
<td>0.83</td>
<td>0.76</td>
</tr>
<tr>
<td>Model</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pre-reform</td>
<td>0.80</td>
<td>0.20</td>
</tr>
<tr>
<td>Post-reform</td>
<td>0.72</td>
<td>0.21</td>
</tr>
</tbody>
</table>
The model-simulated correlation of investment with output increases after the reform, although the model is not able to capture the sharp rise in the correlation as we observe in the data. The data shows that the correlation of trade balance to GDP ratio turns from acyclical to strongly counter-cyclical. Although our model shows that trade balance to GDP ratio has a negative correlation with output, and the magnitude of the correlation increases in the post reform period, but it does not become strongly countercyclical after the reform. The correlation of consumption with output declines, whereas as it increases in the data after the reform.

6.1 Discussion of the results

The open economy framework, when calibrated to Indian data, supports the main prediction of rising relative consumption volatility with financial inclusion. Broadly, the model simulated moments show similar patterns observed in the closed economic framework, except for a marginal rise in the output volatility in the post reform period.

One plausible reason for the open economy set up to show similar trends in the volatility and correlation of the key macroeconomic indicators as in the closed economy scenario is that financial deepening, in our model, works through the household channel. Under strong permanent income shock, relative to transitory income fluctuations, Ricardian households behave in a similar manner in both closed and open economy set up. However, the extent of fluctuations is higher in open economy. In response to permanent income shock, in an open economy, households can even raise current consumption more by using the fund borrowed against the future income. Hence fluctuation in consumption is even higher than the closed economy scenario. Financial inclusion, in this set up results in larger fluctuations in aggregate consumption. Sharp rise in consumption volatility, with relatively smaller decline in investment volatility causes a marginal rise in post reform output fluctuations. Hence, the open and closed economy set up show qualitatively similar results.

In this open economy framework, consumers transact an internationally traded bond, which is the source of capital flows in the economy. A bulk of literature have explored macroeconomic effects of the interaction between financial openness and domestic financial development through firm borrowing channel (Aghion et al., 2004; 2010). Incorporating borrowing by firm in the model may provide an additional channel for the interaction between financial development and financial liberalisation to affect output and investment.
This figure depicts an index of capital account openness based on the “Annual Report on Exchange Arrangements and Exchange Restrictions” of the IMF (Chinn and Ito, 2008). This figure compares the index of capital account openness for India with the emerging economy mean. The set of emerging economies includes countries in Table 1 of the paper, except Taiwan.

![De Jure Financial Integration](image)

However, in spite of the fact that India started liberalising capital account since 1991, the pace and the extent of easing restrictions on capital flows remained low compared to other emerging economies. The access to foreign capital by Indian households and firms are still limited due to wide array of capital control measures existing in the country. The de-jure measure of capital account openness based on Chinn-Ito index shows that India is relatively closed compared to other large emerging economies (Patnaik and Shah, 2012) (see Figure 6). Households in India are not allowed to borrow abroad. There are a number of restrictions on firm borrowing and both macro and firm level data indicates low exposure of Indian firms to foreign capital. 

Along with domestic financial deepening, opening up of the capital account, or financial liberalisation has been a major component of the reforms in India since 1991. However, the access to foreign capital by Indian households and firms have remained limited. Households and banks in India are not allowed to borrow abroad. As far as borrowing by firms are concerned, Indian firms access foreign capital through two channels to leverage their operations. These are Foreign Direct Investment (FDI) and foreign borrowings. FDI in India (net inflows) has grown from USD 0.59 billion in 1993-94 to USD 30.76 billion in 2013-14 (Economic Outlook, Centre for Monitoring Indian Economy). However, the net FDI inflows in India accounts for only 1.78% of GDP in 2013-14. The share of net FDI inflows in India in total investment amounts to 5.24% in 2013-14. To compare with other emerging economies, for instance, net FDI inflows in Brazil in 2013 has been USD 80.84 billion, which is more than double the FDI inflows in India, while the net FDI inflows in China in 2013 has been USD 347.85, which is more than 11 times larger the FDI flows in India (World Development Indicators). Looking deep into the firm-level data base, only 623 firms are found to have foreign promoter (ownership) in a base of 26,725 companies at the end of 31st March, 2014 (Prowess, Centre for Monitoring Indian Economy). India
the low level of access to foreign capital by Indian households and firms, an open economy set up through the financial channel may not be appropriate to replicate the post-reform business cycle stylised facts in India.

India liberalised current account at a faster pace than capital account. Explicitly modelling the current account incorporating home and foreign goods in consumption and investment as in Mendoza (1995); Kose and Yi (2006), would provide an additional channel of trade liberalisation to affect macroeconomic volatility and cyclicality of various indicators with output.

7 Conclusion

Emerging economies have been seen to witness an increase in consumption volatility relative to output volatility after financial development. This behaviour appears puzzling since traditional models and evidence from advanced economies suggests that consumption should become smoother with increase in the access to financial services.

A distinguishing feature of the developing economies is that a large share of the population does not have access to finance. In the last two decades, these economies have experienced reforms in the financial sector giving greater access to financial services for households and firms. Yet, these economies experienced an increase in consumption volatility relative to output volatility in the post-reform period. This paper addresses this empirical puzzle. This puzzle can be explained in a model featuring credit constraints and shocks to trend growth of productivity. The model predicts that relative consumption volatility will rise when more consumers can smooth consumption.

The model, when simulated for India before and after an increase in financial development broadly replicates the rise in the relative consumption volatility as observed in the data. Most of the other empirical regularities observed from the data are also replicated by this model.

Our benchmark model represents a closed economy, and the concept of financial development is limited to household’s access to financial services.

---

holds a stock under foreign borrowings of USD 53.92 billion in 2012-13 and 2013-14. The net inflows of foreign borrowings has accounted for only 0.63% of GDP in 2013-14. Again in a sample of 26,725 firms in the Prowess database, only a total of 642 companies are found to have foreign borrowings over the years, while only 464 companies have executed for the financial year 2013-14.
Our model assumes that the household sector is the sole channel for the financial development to work. This is one plausible reason for our model’s weak performance in replicating the business cycle patterns with respect to investment. By including credit-constrained firms in our framework, one can examine the role of financial development further. Extending the model with borrowings by firms will help in understanding how increase in households’ access to finance affects consumption-smoothing behaviour when production and demand for resources are subject to firm’s access to finance.

Finally, the open economy framework following Aguiar and Gopinath (2007) assumes that consumers transact an internationally traded bond, which is the source of capital flows in the economy. A bulk of literature have explored macroeconomic effects of the interaction between financial openness and domestic financial development through firm borrowing channel (Aghion et al., 2004, 2010). However, a wide array of capital control measures existing in India (Patnaik and Shah, 2012) restricts access of Indian households and firms to foreign capital. Again, India liberalised current account at a faster pace than capital account. Hence an open economy framework, capturing trade liberalisation following Mendoza (1995); Kose and Yi (2006) may help in improving the fit of the model in the open economy framework.

Further, differentiating between agricultural and non-agricultural goods in the consumption basket may help us to capture the effect of structural shifts away from agriculture to non-agriculture on the post-reform stylised facts.
A Appendix A

<table>
<thead>
<tr>
<th>Region &amp; reform date</th>
<th>Span of data</th>
<th>Source of reform date</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Start date</td>
<td>End date</td>
</tr>
<tr>
<td>Chile</td>
<td>1974</td>
<td>2010</td>
</tr>
<tr>
<td>Colombia</td>
<td>1968</td>
<td>2010</td>
</tr>
<tr>
<td>Mexico</td>
<td>1978</td>
<td>2011</td>
</tr>
<tr>
<td>Peru</td>
<td>1989</td>
<td>2011</td>
</tr>
<tr>
<td>Indonesia</td>
<td>1978</td>
<td>2011</td>
</tr>
<tr>
<td>Malaysia</td>
<td>1970</td>
<td>2011</td>
</tr>
<tr>
<td>Philippines</td>
<td>1958</td>
<td>2011</td>
</tr>
<tr>
<td>Korea</td>
<td>1953</td>
<td>2010</td>
</tr>
<tr>
<td>Taiwan</td>
<td>1981</td>
<td>2011</td>
</tr>
<tr>
<td>Thailand</td>
<td>1950</td>
<td>2011</td>
</tr>
<tr>
<td>Turkey</td>
<td>1989</td>
<td>2010</td>
</tr>
<tr>
<td>Poland</td>
<td>1981</td>
<td>2011</td>
</tr>
<tr>
<td>Hungary</td>
<td>1971</td>
<td>2011</td>
</tr>
<tr>
<td>South Asia</td>
<td></td>
<td></td>
</tr>
<tr>
<td>India: 1992</td>
<td>1951</td>
<td>2012</td>
</tr>
<tr>
<td>Africa</td>
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<td></td>
</tr>
<tr>
<td>South Africa: 1994</td>
<td>1950</td>
<td>2011</td>
</tr>
</tbody>
</table>

Given the availability of data for Peru, we compare relative consumption volatility before and after 2000.

B Appendix B

B.1 Solution of the log-linearised system of equations using method of undetermined coefficients

Log-linearisation of the system of equations around the steady state, and substituting the log-linearised expression of $\Omega_i$ in that of the Euler equation.
yields,
\[
\tilde{c}_t^R = \left(1 - \frac{\lambda \alpha}{1 - \lambda}\right) \frac{y^*}{c_{R*}} + \left[\left(1 - \frac{\lambda \alpha}{1 - \lambda}\right) \frac{\mu_g k^*}{\beta c_{R*}} - \frac{\lambda (1 - \alpha) (1 - \delta) k^*}{1 - \lambda} \frac{c^*}{c_{R*}}\right] \tilde{k}_t^R - \frac{\mu_g k^R_{cR*}}{c_{R*}} \tilde{k}_{t+1}^R - \frac{\mu_g k^R_{cR*}}{c_{R*}} \tilde{g}_t,
\]
\[0 = E_{t-1} \left[\tilde{c}_{t+1}^R - \tilde{c}_t^R + A(a_t - \alpha \tilde{k}^R_t + \alpha \tilde{g}_t)\right];\]
\[A = 1 - \frac{\beta (1 - \delta)}{\mu_g},\]
\[a_t = \rho_a a_{t-1} + \epsilon_t^a,\]
\[\tilde{g}_t = \rho_g \tilde{g}_{t-1} + \epsilon_t^g;\]
\[\tilde{g}_t = \ln \left(\frac{g_t}{\mu_g}\right),\]
\[(B.1)\]

where the cyclical component of a variable \(x_t\) is defined as \(\tilde{x}_t = \ln x_t - \ln x^*\) and \(x^*\) denotes the steady state value of \(x_t^*\). The steady state growth rate of labour productivity is the long term average trend growth rate \(\mu_g\). The last two equations in the system of equations \((B.1)\) yield volatility of transitory and permanent income shocks as \(\sigma^2_a = \frac{\sigma^2_{\epsilon_a}}{1 - \rho^2_a}\) and \(\sigma^2_g = \frac{\sigma^2_{\epsilon_g}}{1 - \rho^2_g}\).

The solution of the system of equations \((B.1)\) takes the form
\[
\tilde{k}_{t+1}^R = a_1 \tilde{k}_t^R + b_1 a_t + d_1 \tilde{g}_t
\]
\[
\tilde{c}_t^R = a_2 \tilde{k}_t^R + b_2 a_t + d_2 \tilde{g}_t.
\]
\[(B.2)\]

The unknown parameters \((a_1, b_1, d_1, a_2, b_2, d_2)\) are functions of \((\beta, \alpha, \delta, \lambda, \mu_g)\) and are solved using the method of undetermined coefficients. The solution \((B.2)\) of the dynamic system indicates that each endogenous variable in time \(t\) is a linear function of the state variables \((k^R_t, a_t, g_t)\). For the system to satisfy transversality condition, i.e., convergence of the system to the steady state over time, \(k^R_t\) must converge to zero following a shock. That is, \(a_1\) must satisfy the condition \(a_1 < 1\).

Substituting solution \((B.2)\) in the first equation in the system of equations \((B.1)\), we have
\[
a_2 \tilde{k}_t^R + b_2 a_t + d_2 \tilde{g}_t = \left(1 - \frac{\lambda \alpha}{1 - \lambda}\right) \frac{y^*}{c_{R*}} + \left[\left(1 - \frac{\lambda \alpha}{1 - \lambda}\right) \frac{\mu_g k^*}{\beta c_{R*}} - \frac{\lambda (1 - \alpha) (1 - \delta) k^*}{1 - \lambda} \frac{c^*}{c_{R*}}\right] \tilde{k}_t^R - \frac{\mu_g k^R_{cR*}}{c_{R*}} [a_1 \tilde{k}_t^R + b_1 a_t + d_1 \tilde{g}_t] - \frac{\mu_g k^R_{cR*}}{c_{R*}} \tilde{g}_t
\]

\(8\)We use the approximation \(x_t = e^{\tilde{x}_t} x^* \approx (1 + \tilde{x}_t) x^*\) to log-linearise the equations of the model.
Re-arranging the terms yields

\[
\begin{align*}
    a_2 &= \left( \frac{1 - \lambda}{1 - \alpha} \right) \mu_g k^{R*} - \frac{\lambda (1 - \delta) (1 - \alpha) k^{R*}}{1 - \lambda} c^{R*} - \frac{\mu_g k^{R*}}{c^{R*}} a_1 \right] \quad (B.3) \\
    b_2 &= \left( \frac{1 - \lambda}{1 - \alpha} \right) y^* c^{R*} - \mu_g k^{R*} \quad (B.4) \\
    d_2 &= -\left[ \frac{\mu_g k^{R*}}{c^{R*}} d_1 + \frac{\mu_g k^{R*}}{c^{R*}} b_1 \right] \quad (B.5)
\end{align*}
\]

where the steady state value of consumption of a Ricardian household, and the steady state value of the output are respectively

\[
\begin{align*}
    c^{R*} &= \left( \frac{1 - \lambda}{1 - \alpha} \right) \left( 1 - \lambda \right)^{1 - \alpha} \mu_g^\alpha - \left( \mu_g - 1 + \delta \right) k^{R*} \\
    y^* &= \left( 1 - \lambda \right) \left( 1 - \alpha \right) \left( k^{R*} \right)^{1 - \alpha} \mu_g^\alpha \\
\end{align*}
\]

Given the steady state value of capital stock of a Ricardian household is,

\[
\begin{align*}
    k^{R*} &= \left[ \left( \frac{1 - \lambda}{1 - \alpha} \right) (1 - \lambda)^{1 - \alpha} \mu_g^\alpha \right]^{1/\alpha} \\
\end{align*}
\]

The steady state expression of stock of capital of a Ricardian household is derived from the Euler equation and the expression of gross return to capital given in the equation system \( \{19\} \) assuming that in the steady state, all variables normalised by the labour productivity of the previous period remain constant along the steady state, i.e., \( \ddot{k}_t = \tilde{k}^{R*} = k^{R*} \), and \( \ddot{c}_t = \tilde{c}^{R*} = c^{R*} \). The steady state growth rate of labour productivity is the long-run average trend growth rate \( \mu_g \). Also, the steady state is free of any transitory movement in the total factor productivity, hence, \( a_t = a_{t+1} = 0 \). Given the value of \( k^{R*} \), the steady state value of consumption \( c^{R*} \) is derived from the resource constraint equation in \( \{19\} \). It then follows from equations \( \{11\}, \{14\}, \{12\} \) that the steady state consumption of the liquidity constrained households and the total consumption are respectively \( c^{L*} = \alpha y^* \) and \( c^* = (1 - \lambda) c^{R*} + \lambda c^{L*} \).

Substituting the solution \( \{B.2\} \) in the second equation of \( \{B.1\} \), and making the use of \( E_{t-1} a_t = \rho_a a_{t-1} \) and \( E_{t-1} \tilde{g}_t = \rho_g \tilde{g}_{t-1} \) yields

\[
\begin{align*}
    a_2 - a_2 a_1 - A \alpha a_1 &= 0 \quad (B.6) \\
    b_2 (1 - \rho_g) - a_2 b_1 + A \rho_a - A \alpha b_1 &= 0 \quad (B.7) \\
    d_2 (1 - \rho_g) - a_2 d_1 - A \alpha d_1 + A \alpha \rho_g &= 0 \quad (B.8)
\end{align*}
\]

Again, substituting expression \( \{B.4\} \) in the expression \( \{B.8\} \), and rearranging the terms yields a quadratic equation in \( a_1 \),

\[
a_1^2 - \gamma_1 a_1 + \gamma_2 = 0 \quad (B.9)
\]

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where

\[ \gamma_1 = 1 + \left( \frac{1 - \lambda \alpha}{1 - \lambda} \right) \frac{1}{\beta} - \frac{\lambda(1 - \alpha)(1 - \delta)}{(1 - \lambda)\mu_g} + A\alpha \frac{c^{R*}}{\mu_g k^{R*}} \]

\[ \gamma_2 = \left( \frac{1 - \lambda \alpha}{1 - \lambda} \right) \frac{1}{\beta} - \frac{\lambda(1 - \alpha)(1 - \delta)}{(1 - \lambda)\mu_g} \]

(B.10)

(B.11)

The solution of the above quadratic equation is

\[ a_1 = \frac{\gamma_1}{2} \pm \sqrt{\left( \frac{\gamma_1}{2} \right)^2 - \gamma_2} \]

(B.12)

If there is no friction in the credit market so that the entire population can access financial services and hence smooth consumption, then \( \lambda = 0 \). Then the product of the two roots in expression (B.12) is \( \gamma_2 = 1/\beta > 1 \), where the value of \( \gamma_2 \) is obtained by evaluating expression (B.11) at \( \lambda = 0 \). Again, as \( \lim_{\lambda \to 0} \gamma_2 \to \infty \). Hence, the product of the two roots as in the expression (B.12) \( \gamma_2 \) is always greater that 1, irrespective of the value of \( \lambda \). Therefore it follows that the larger one must exceed 1 and only the smaller one can possibly satisfy the convergence condition \( a_1 < 1 \). Hence, the solution of the quadratic equation (B.9) is

\[ \hat{a}_1 = \frac{\gamma_1}{2} - \sqrt{\left( \frac{\gamma_1}{2} \right)^2 - \gamma_2}. \]

(B.13)

Given this solution of \( a_1 \), from equations (B.4), (B.5), (B.6), (B.7), (B.8), (B.9) and making use of the steady state values \( (k^{R*}, c^{R*}, y^*) \), one can solve for \((a_2, b_1, b_2, d_1, d_2)\) as

\[ \hat{a}_2 = \left[ \left( 1 - \frac{\lambda \alpha}{1 - \lambda} \right) \frac{\mu_g k^{R*}}{\beta c^{R*}} - \frac{\lambda(1 - \delta)(1 - \alpha) k^{R*}}{1 - \lambda} \frac{c^{R*}}{c^{R*}} - \frac{\mu_g k^{R*}}{c^{R*}} \hat{a}_1 \right] \frac{\mu_g k^{R*}}{c^{R*}} \]

\[ \hat{b}_1 = \frac{(1 - \rho_g) \left( 1 - \frac{\lambda \alpha}{1 - \lambda} \right) y^*}{\mu_g k^{R*}} + \hat{a}_2 + A\lambda \]

\[ \hat{b}_2 = \left( 1 - \frac{\lambda \alpha}{1 - \lambda} \right) \frac{y^*}{c^{R*}} - \frac{\mu_g k^{R*}}{c^{R*}} \hat{b}_1 \]

\[ \hat{d}_1 = \frac{A\alpha \rho_g - (1 - \rho_g) \mu_g k^{R*}}{\mu_g k^{R*}} \frac{c^{R*}}{c^{R*}} + \hat{a}_2 + A\alpha \]

\[ \hat{d}_2 = -\frac{\mu_g k^{R*}}{c^{R*}} (1 + \hat{d}_1) \]

(B.14)
Given the total consumption of the economy as in equation (12), and making use of the equation (11) and the fact that \( W_t = \alpha Y_t \) implying \( \tilde{c}_t^L = \tilde{y}_t \), one can arrive at the volatility of consumption relative to output as,

\[
\frac{\sigma^2_{\tilde{c}_t^L}}{\sigma^2_{\tilde{y}_t}} = \left( \frac{c^R}{c^*} \right)^2 (1 - \lambda) \frac{\sigma^2_{\tilde{c}_t^R}}{\sigma^2_{\tilde{y}_t}} + \left( \frac{c^L}{c^*} \right)^2 \lambda^2.
\]  

(B.15)

Here the fluctuations in a Ricardian household’s consumption and that in total output are respectively,

\[
\sigma^2_{\tilde{c}_t^R} = \left[ \frac{a_2^2 b_1^2}{1 - a_1^2} + \frac{b_2^2}{1 - a_1^2} \right] \sigma^2_a + \left[ \frac{a_2^2 d_1^2}{1 - a_1^2} + \frac{d_2^2}{1 - a_1^2} \right] \sigma^2_\gamma, \\
\sigma^2_{\tilde{y}_t} = \left[ 1 + \frac{(1 - \alpha)^2 b_1^2}{1 - a_1^2} \right] \sigma^2_a + \left[ \alpha^2 + \frac{(1 - \alpha)^2 d_1^2}{1 - a_1^2} \right] \sigma^2_\gamma,
\]

(B.16)

where the value of the coefficients of \( \sigma^2_a \) and \( \sigma^2_\gamma \) in equations B.16 can be expressed in terms of the parameters derived in B.14.

We demonstrate a proof of Proposition (1) through a numerical analysis. The volatility of consumption and output are computed using the expressions in (B.10), (B.11), (B.13) and (B.14) evaluated with values of deep parameters, the steady state growth rate and the persistence parameters of technology and growth shock structures used in the simulation exercise in section 5. The values of the parameters in the numerical exercise are \( \beta = 0.98, \alpha = 0.7, \delta = 0.05, \mu_y = 1.047, \rho_a = 0.495, \) and \( \rho_\gamma = 0.261. \)

B.2 Proof of Proposition (1) (i)

Log-linearisation of \( c_t^L = W_t = \alpha y_t \) yields \( \tilde{c}_t^L = \tilde{y}_t \) Hence \( \sigma^2_{\tilde{c}_t^L} = \sigma^2_{\tilde{y}_t} \) proves Proposition (1) (i).

B.3 Proof of Proposition (1) (ii)

Under the assumption of the absence of growth shock, \( \sigma^2_\varepsilon = 0, \) \( \sigma^2_\gamma > 0, \) from expressions in (21), the relative consumption volatility of a Ricardian household with respect to output volatility is obtained as

\[
\frac{\sigma^2_{\tilde{c}_t^R}}{\sigma^2_{\tilde{y}_t}} = \frac{\left[ \frac{a_2^2 b_1^2}{1 - a_1^2} + \frac{b_2^2}{1 - a_1^2} \right]}{1 + \frac{(1 - \alpha)^2 b_1^2}{1 - a_1^2}}.
\]  

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We evaluate this ratio at two values of $\lambda$. In one scenario, $\lambda = 0.9$ that is the share of liquidity-constrained households is very high. In the second scenario, the economy is populated only by the Ricardian households, i.e., $\lambda = 0$. Given the value of relative volatility of consumption of a Ricardian household with respect to output volatility, the volatility of total consumption with respect to output volatility $\sigma^2_{\tilde{c}} / \sigma^2_{\tilde{y}}$ can be computed from equation (20) under two alternative state of financial development. The relative consumption volatilities are shown in Table B.1.

### Table B.1 Relative consumption volatility under transitory income shock

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>$\sigma^2_{\tilde{c}} / \sigma^2_{\tilde{y}}$</th>
<th>$\sigma^2_{\tilde{c}R} / \sigma^2_{\tilde{y}}$</th>
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</thead>
<tbody>
<tr>
<td>0.900</td>
<td>0.745</td>
<td>0.807</td>
</tr>
<tr>
<td>0</td>
<td>0.406</td>
<td>0.406</td>
</tr>
</tbody>
</table>

The results indicate that as long as the economy contains a share of population of Ricardian type who can smooth consumption, the consumption volatility relative to output volatility is less than one under a transitory income shock. Also note that when $\lambda = 0$, that is the economy is populated by only Ricardian households, relative volatility of total consumption with respect to output is same as the relative consumption volatility of the Ricardian household.

### B.4 Proof of Proposition 1 (iii)

Under the assumption of the absence of transitory income shock, $\sigma^2_{\epsilon_a} = 0$, $\sigma^2_{\epsilon_g} > 0$, from expressions in (21), the relative consumption volatility of a Ricardian household with respect to output volatility is obtained as

$$
\frac{\sigma^2_{\tilde{c}R}}{\sigma^2_{\tilde{y}}} = \left[ \frac{a_1^2 d_1^2}{1-a_1^2} + d_2^2 \right] \left[ \alpha^2 + \frac{(1-\alpha)^2 d_1^2}{1-a_1^2} \right].
$$

We evaluate this ratio for a range of values of $\lambda$. The highest value of $\lambda = 0.9$ corresponds to the scenario when the share of liquidity-constrained households is very high. The lowest value that $\lambda$ takes is 0. This scenario represents an economy with a matured financial system so that it is populated only by the Ricardian households. We also consider the values of $\lambda$ used in the simulation exercise in section 5 to evaluate relative consumption volatility of Ricardian households and that of the entire economy. Given the value of
relative volatility of consumption of a Ricardian household with respect to output volatility, the volatility of total consumption with respect to output volatility $\frac{\sigma_c^2}{\sigma_y^2}$ can be computed from equation (20) under alternative states of financial development. The relative consumption volatilities are shown in Table B.2.

Table B.2 Relative consumption volatility under permanent income shock

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>$\frac{\sigma_c}{\sigma_y}$</th>
<th>$\frac{\sigma_R}{\sigma_y}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.900</td>
<td>1.493</td>
<td>0.834</td>
</tr>
<tr>
<td>0.786</td>
<td>1.992</td>
<td>1.018</td>
</tr>
<tr>
<td>0.500</td>
<td>3.086</td>
<td>1.897</td>
</tr>
<tr>
<td>0.331</td>
<td>3.556</td>
<td>2.585</td>
</tr>
<tr>
<td>0</td>
<td>4.223</td>
<td>4.223</td>
</tr>
</tbody>
</table>

The results indicate that the relative consumption volatility with respect to output volatility of a Ricardian household always exceeds 1. The relative volatility of total consumption with respect to output volatility depends on the level of financial development. The relative consumption volatility may fall below 1 if the economy consists of a large fraction of liquidity-constrained households.

B.5 Proof of Proposition 1 (iv)

From Table B.2, it is evident that the volatility of consumption of a Ricardian household and that of the total consumption with respect to output volatility increases as share of liquidity constrained households in the economy declines.

C Appendix C

Normalising all variables by the permanent component of productivity at period $t-1$, we arrive at the following detrended system of the open economy.
The first order conditions for the Ricardian households are as follows,

\[
\frac{1}{c^R_t} = \Lambda_t
\]

\[
\Lambda_t g_t \left[ 1 + \phi \left( \frac{k^R_{t+1} g_t}{k^R_t} - \mu_g^\ast \right) \right] = \beta \Lambda_{t+1} \left[ R^K_{t+1} + (1 - \delta) - \frac{\phi}{2} \left( \frac{k^R_{t+2} g_{t+1}}{k^R_{t+1}} - \mu_g^\ast \right)^2 \right]
\]

\[
+ \phi \left( \frac{k^R_{t+2} g_{t+1}}{k^R_{t+1}} - \mu_g^\ast \right) \left( \frac{k^R_{t+2} g_{t+1}}{k^R_{t+1}} - \mu_g^\ast \right)
\]

(C.1)

where \( R^K_{t+1} = (1 - \alpha)(1 - \lambda)^{1-\alpha} e^{a_{t+1}} k^R_{t+1} - a_{t+1}, \) and,

\[
\Lambda_t \left[ \frac{g_t}{1 + R_t} \right] = \beta E_t \Lambda_{t+1}.
\]

(C.2)

The budget constraint of a Ricardian household and a liquidity-constrained household are respectively as follows,

\[
c^R_t + i^R_t + b^R_t - g_t \frac{b^R_{t+1}}{1 + R_t} = R^K_t k^R_t + \omega_t
\]

(C.3)

\[
i^R_t = g_t k^R_{t+1} - (1 - \delta) k^R_t + \frac{\phi}{2} \left( g_t k^R_{t+1} - \mu_g^\ast \right)^2 k^R_t
\]

(C.4)

\[
c^L_t = \omega_t
\]

(C.5)

where

The normalised return to labour is expressed as

\[
\omega_t = \alpha e^{a_t} (1 - \lambda)^{1-\alpha} k^R_t^{-\alpha} g_t^\ast.
\]

(C.7)

The aggregate consumption, investment, capital stock and debt are respectively,

\[
c_t = \lambda c^R_t + (1 - \lambda) c^L_t, \quad i_t = (1 - \lambda) i^R_t, \quad k_t = (1 - \lambda) k^R_t, \quad b_t = (1 - \lambda) b^R_t
\]

(C.8)

The output produced in the economy and the interest rate on bond are respectively the following

\[
y_t = e^{a_t} [(1 - \lambda) k^R_t]^{1-\alpha} g_t^\ast
\]

(C.9)

\[
R_t = R^\ast + \psi (e^{b_{t+1} - \tau} - 1)
\]

(C.10)
respectively. The economy-wide resource constraint is

$$c_t + i_t + tb_t = y_t \quad \text{(C.11)}$$

where

$$tb_t = b_t - g_t \frac{b_{t+1}}{1 + R_t} \quad \text{(C.12)}$$

With the initial capital stock $k_0$, the competitive equilibrium is defined as a set of prices and quantities $(R^K_t, \omega_t, R_t, y_t, c^R_t, c^I_t, i^R_t, k^R_t, b^R_t, c_t, i_t, k_t, b_t, tb_t)$, given the sequence of shocks to the transitory component of TFP and the growth rate of the permanent component of TFP, that solves the maximisation problem of the households, optimisation by the firms and satisfies the resource constraint of the economy.

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