New Keynesian DSGE Models: Building Blocks

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Features of Blachard-Kiyotaki Model

- A static model; has commodity, labor and money markets. No bonds market; hence no interest rate.
- By Walras law, two independent market clearing conditions, determining P and W.
- Single generic product, differentiated. The product market: monopolistically competitive.
- Producers are price setters. Later, this feature is exploited to consider partial price rigidity facing price setting firms.
- Consumers/households derive utility from consumption of different varieties and holding money. It is in the tradition of MIU (Money-in-the-Utility) approach.
- Plus, they derive disutility from work.

Consumption and Money Demand Choices

Utility Function: $\tilde{U}_t^h = B(C_t^h)^\gamma \left(\frac{M_{dt}^h}{P_t}\right)^{1-\gamma}$, where $C_t^h \equiv \int_0^1 (C_{jt}^h)^{\frac{\eta-1}{\eta}} dj$.

$$0 < \gamma < 1 < \eta$$

Budget Constraint: $\int_0^1 P_{jt} C_{jt}^h dj + M_{dt}^h \leq E_t^h \equiv Y_t^h + M_{t-1}^h$, where

$$Y_t^h = W_t L_t^h + \Pi_t^h; \quad M_{t-1}^h = \bar{M}^h.$$

Will be satisfied with equality. Solutions:

$$C_{jt}^h = \gamma \left(\frac{P_{jt}}{P_t}\right)^{-\eta} \frac{E_t^h}{P_t}; \; C_t^h = \gamma \frac{E_t^h}{P_t}; \; \frac{M_{dt}^h}{P_t} = (1-\gamma)\frac{E_t^h}{P_t}$$

Indirect Utility: $\tilde{U}_t = B\gamma^\gamma (1-\gamma)^{1-\gamma} \frac{E_t^h}{P_t} = \frac{E_t^h}{P_t}$, by normalizing $B\gamma^\gamma(1-\gamma)^{1-\gamma} = 1$.

Leisure-Consumption Choice, Labor Supply Function, Technology & Firm Behavior

- "Total" Utility Function: $\frac{E_t^h}{P_t} - \frac{(N_t^h)^{1+\Phi}}{1+\Phi}$; Constraint: $\frac{E_t^h}{P_t} = \frac{\Pi_t^h + W_t N_t^h + \bar{M}^h}{P_t}$
- FOC $(N_t^h)^{\Phi} = \frac{W_t}{P_t} \Rightarrow N_t^h = \left(\frac{W_t}{P_t}\right)^{1/\Phi}$, Labor Supply Function.
- Technology: $Y_{jt} = L_{jt}$. $\mathrm{MC}_t = W_t$.
- Firm Behavior: Firm j faces a constant-elasticity demand function:

$$C_{jt} = \sum_h C_{jt}^h = \gamma \left(\frac{P_{jt}}{P_t}\right)^{-\eta} \frac{E_t^h}{P_t} = \gamma \left(\frac{P_{jt}}{P_t}\right)^{-\eta} \frac{E_t}{P_t} \equiv A_t P_{jt}^{-\eta}.$$

- $\boxed{\text{FOC: } P_{jt} = \frac{\eta}{\eta-1} W_t}$, $\Leftrightarrow \frac{P_{jt}}{P_t} = \frac{\eta}{\eta-1} \frac{W_t}{P_t}$.
- Note: $\frac{\eta}{\eta-1}$ is the Mark up.

Solution

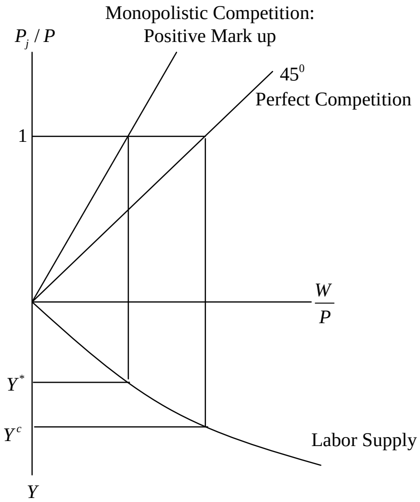
- Aggregating, noting that W/P is the "real marginal cost", mc,

$$\frac{P_{jt}}{P_t} = \frac{\eta}{\eta-1}\frac{W_t}{P_t} = \frac{\eta}{\eta-1}\text{mc} = \frac{\eta}{\eta-1}N^{\Phi} = \frac{\eta}{\eta-1}Y^{\Phi}$$

- In Equilibrium: $P_{jt} = P_t$. Substituting it above solves output and employment.
- In turn this solves real wage.
- Money Market Clearing: $M_t^d = \bar{M}$, i.e. $(1-\gamma)E_t = \bar{M}$, i.e., $(1-\gamma)(P_t Y_t + \bar{M}) = \bar{M}$. It solves P_t. Given W_t/P_t, W_t is solved.
- Called Flexi-Price Equilibrium

Two Properties of the Flexi-Price Equilibrium

- Money is neutral.
- Equilibrium is suboptimal: Flexi-price equilibrium employment is less than “true”, competitive-level full-employment.



NKPC: Introduction

$\boxed{\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + e_t}$, where $x_t \equiv y_t - y_t^*$ is the deviation of log output from its "flexi" level and e_t is a cost push, inflation shock or price shock. Typically called: a cost shock.

- Unlike the traditional Phillips curve, output does NOT depend, besides π_t, on expected inflation in t given information at $t-1$ but expected inflation for the *next period.*
- We can expand it as:

$$\pi_t = \kappa \sum_{i=0}^{\infty} \beta^i E_t x_{t+k} + \sum_{i=0}^{\infty} \beta^i E_t e_{t+k}.$$

  It resembles an asset price equation, saying that inflation today is like the "price" of current and future output gaps. Inflation is entirely forward looking.
- As $E_t p_{t+1} \uparrow$, AS curve shifts to the left, i.e., future expected inflation affects current output. This doesn't happen with the traditional Phillips curve.

NKPC Introduction Cont.

- Early empirical evidence: supportive. More recent ones are more sceptical. Still a lot of estimation is going on. Look at various free issues of the new Web-journal, **E-conomics.**
- One of the earliest empirical works: Roberts (1995), supportive.
- Using single-equation GMM, Gali and Gertler (1999) Gali, Gertler and Lopez-Salido (2001) also find empirical support.
- So was Sbordone (2002) in a slightly different setup.
- Recent empirical work includes those of Matheron-Maury (2004), McAdam-Willman (2004), Roberts (2005) and Nelson and Lee (2007).
- 3 micro-foundations: leading to the same equation.

Calvo (1983) Most Widely Used

- Consider a monopolistically competitive industry as in the Blanchard-Kiyotaki model.
- At any t, only a fraction of firms, $1-b$, are able to change prices. The rest are not.
- Thus a firm, while choosing its price, takes into consideration that its chosen price may remain the same in the next period with prob $= b$. The same price may prevail two periods from now with prob $= b^2$ and so on.
- Let z_t be its price chosen. A firm which is able to choose its price today minimizes the loss function:

$$
\begin{aligned}
L(z_t) &= \sum_{k=0}^{\infty} (b\beta)^k E_t (z_t - p^*_{t+k})^2 \\
&= (z_t - p^*_t)^2 + b\beta(z_t - p^*_{t+1})^2 + b^2\beta^2(z_t - p^*_{t+2})^2 + \cdots
\end{aligned}
$$

Calvo Continued

- Nice Thing: Price stickiness is captured by the parameter b and it appears a time-discount factor!!!
- FOC $z_t = (1-b\beta)\sum_{k=0}^{\infty}(b\beta)^k E_t p_{t+k}^* = (1-b\beta)\sum_{k=0}^{\infty}(b\beta)^k E_t p_{t+k}^*$. FOC can be written as $\boxed{z_t = b\beta E_t z_{t+1} + (1-b\beta)p_t^*.}$ We have $\boxed{p_t = bp_{t-1} + bz_t.}$ Using this, eliminate z_t and $E_t z_{t+1}$ above. We get $\pi_t = \beta E_t \pi_{t+1} + \frac{(1-b)(1-b\beta)}{b}(p_t^* - p_t)$.
- $p_t^* = \mu + MC_t = \mu + w_t \Rightarrow \mu = -(w_t - p_t^*) = -\Phi y_t^*$.
- $p_t^* - p_t = \mu + MC_t - p_t = -\Phi y_t^* + w_t - p_t = -\Phi y_t^* + \Phi y_t = \Phi x_t$.

  Key: Real marginal cost rises with output gap.

Rotemberg (1982)

- Firms face an adjustment cost of changing prices from one period to the next, while they recognize that $p_t^* = \mu + w_t$.
- Each firm minimizes $E_0 \sum_{t=0}^{\infty} \beta^t \left[(p_t - p_t^*)^2 + c(p_t - p_{t-1})^2 \right]$
- FOC: $\beta^t[2(p_t - p_t^*) + 2c(p_t - p_{t-1})] - \beta^{t+1} c(p_{t+1} - p_t) = 0$, simplifying to
- $\pi_t = \beta_t \pi_{t+1} + \frac{1}{c}(p_t^* - p_t)$.
- Already seen: $p_t^* - p_t = \Phi x_t$.
- Hence $\pi_t = \beta E_t \pi_{t+1} + \frac{\Phi}{c} x_t$

Taylor (1979)

It leads to an equation, which is more general than what is normally specified.

- $w_t^1 = \frac{p_t + E_t p_{t+1} + k(y_t + E_t y_{t+1})}{2}$; $w_t^2 = \frac{p_{t-1} + E_{t-1} p_t + k(y_{t-1} + E_{t-1} y_t)}{2}$
- $p_t = (w_t^1 + w_t^2)/2 + \mu$. Normalizing $\mu = 0$, $p_t = (w_t^1 + w_t^2)/2$.
- Substituting and rearranging

$$\pi_t = E_t \pi_{t+1} + (E_{t-1} p_t - p_t) + k(y_t + E_t y_{t+1} + y_{t-1} + E_{t-1} y_t)$$

.

Utility Function

$$\frac{(C_t^h)^{1-\sigma}}{1-\sigma} - \frac{(N_t^h)^{1+\Phi}}{1+\Phi}, \text{ where } C_t^h \equiv \int_0^1 (C_{jt}^h)^{\frac{\eta-1}{\eta}} dj.$$

- Different from Blachard-Kiyotaki's: (1) does not have money demand. Both kinds of New Keynesian DSGE models, of optimal monetary policy and business cycles, have this feature. Because of interest rate being used as the monetary instrument (will be discussed later).
- (2) Concave in C_t^h, satisfies risk-aversion (constant relative risk-aversion exactly.) σ is the risk-aversion and intertemporal substitution parameter (to be seen).
- Concavity $\Rightarrow$ consumption smoothing over time.

Utility Function Cont

- It also implies a slight different trade off in the labor supply decision.
- (Extra) utility loss from one hour of work $= L_t^{\Phi} = N_t^{\Phi}$.
- (Extra) gain in utility terms $= (W_t/P_t)C_t^{-\sigma}$.
- FOC: $N_t^{\Phi} = (W_t/P_t)C_t^{-\sigma}$
- $\Rightarrow \boxed{\frac{W_t}{P_t} = N_t^{\Phi} C_t^{\sigma}, \;\; w_t - p_t = \Phi n_t + \sigma c_t}$
- This may alter the form of the NKPC.

Euler Equation: Consumption-Savings Trade-Off

- Over time, a household maximizes $E_0 \sum_{t=0}^{\infty} \beta^t U_t$, where

$$U_t = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\Phi}}{1+\Phi}$$

- Intertemporal budget constraint: specifies how a family's consumption and asset accumulation are linked.
- Euler equation: $\frac{\beta u'(C_{t+1})}{u'(C_t)} = \frac{1+E_t\pi_{t+1}}{1+i_t}$ i.e. $\beta\left(\frac{C_t}{C_{t+1}}\right)^{\sigma} = \frac{1+E_t\pi_{t+1}}{1+i_t}$.

  $\boxed{\Rightarrow c_t \simeq E_t c_{t+1} - \frac{1}{\sigma}(i_t - E_t\pi_{t+1}).}$
- Called the Expectational IS curve.
- Interest rate is NOT in log.
- +ve relation between c_t and c_{t+1} follows from consumption smoothing.
- -ve effect of real interest rate on consumption follows from the substitution effect between consumption and savings.

Taylor's Rule

- A. Orphanides, "Taylor's Rules," *The New Palgrave Dictionary of Economics*, 2nd Edition, 2008, pp. 200-04
- It is rule saying a central bank should adjust the interest rate in a systematic way based on inflation and macro activity.
- Friedman's $k\%$ money growth rule based on quantity theory. $\hat{M}+\hat{V}=\hat{P}+\hat{y}$, where $\hat{P}$ is inflation rate. Evaluate the country's growth potential, think of a desired inflation rate, have an estimate of $\hat{V}$ depending on growth of financial innovation and then calcuate $k=\hat{M}$.
- Taylor's Rule: $i_t - i^* = \theta_\pi(\pi_t - \pi^*) + \theta_y(y_t - y^*)$ where π^* is the long-run desired rate of inflation, $i^* = r^* + \pi$ (r^* being the natural or equilibrium rate of real interest rate), y_t is log of real GDP.
- Lots of confusion about which is "target" and which is "instrument". Targets are inflation and output gap; interest rate is the instrument.

Taylor's Rule Continued

- Taylor took $r^* = \pi^* = 2$ where "2" means two percent and weights equal to $1/2$. Thus the rule boils down to

$$i_t = 2 + \pi_t + \frac{1}{2}(\pi_t - 2) + \frac{1}{2}(y_t - y^*).$$

- Research shows that the conduct of U.S. monetary policy fitted this pattern quite well from 1965 onwards (except for 1980-82 when the target was non-borrowed reserves), where i_t is the federal funds rate and inflation refers to the rate of change in the GDP deflator.
- It has appeared (since 1993) in many extended forms.
- There are other rules like McCallum's rule, Brainard's rule etc.
- Bank of England's monetary policy seems to fall into a Taylor's pattern, although there are regime shifts. Across different regimes over time, intercept and slope coefficients are different (Edward Nelson, 2001, "UK Monetary Policy 1972-1997: A Guide Using Taylor Rules".
- India .....Ajay Shah, and, NIPFI ???

Monetary Authority's Loss Function

- It is to minimize

$$\frac{1}{2}E_t\sum_{k=0}^{\infty}\beta^k\left[\lambda(\pi_{t+k}^2+x_{t+k}-x^*)^2\right]$$

  where x^* is the gap between efficient output and the flexi-price equilibrium output.
- $\boxed{x^* > 0 \text{ implies an inflationary bias.}}$
- It means the Central Bank targets both output gap and inflation. If, for example, λ is very high, we say that the Bank targets inflation.
- Micro Foundation: Outlined in Walsh (2003, Chapter 11, Appendix). Based on a quadratic approximation of the utility based welfare function.

Cost of Inflation

- Shoe-Leather Cost of holding money: People have to make trips to banks so often that the leather wears out.
- Unanticipated inflation has serious distributional implications: borrowers gain and lenders lose ("unfair").
- Even anticipated inflation has serious distributional implications: fixed wage earners lose a lot, while other don't to a large degree.
- It adversely affects financial and business planning.
- Except the first, the others are not really modeled in the literature yet.